2). slope mirror symmetry.

Δ. M_λ : $x_1^5 + \cdots + x_5^5 - \lambda x_1 \cdots x_5 = 0$ $/\mathbb{F}_q$

smooth proj.

Δ^*. W_λ mirror smooth proj $/\mathbb{F}_q$.

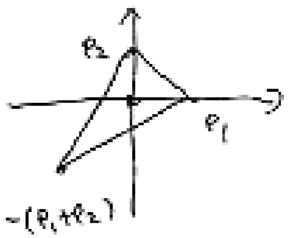
19. Basic example.

$$f(\lambda, x) = x_1 + x_2 + \cdots + x_n + \frac{1}{x_1 \cdots x_n} - \lambda .$$

$$\Delta = \Delta(f) = \langle e_1, e_2, \cdots, e_n, -(e_1 + \cdots + e_n) \rangle$$

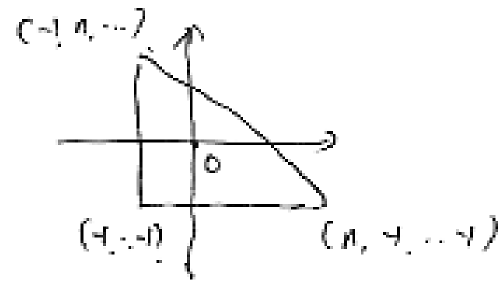
$$\Delta^* = \langle (n, -1, \cdots, -1), \cdots, (-1, -1, \cdots, n), (-1, -1, \cdots, -1) \rangle$$

Δ.



reflexive.

Δ^*.

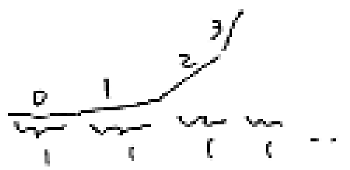


Δ is Fano. Δ^* is NOT Fano $(n>1)$

$f(\lambda, x)$ is Δ-regular, $\Longleftrightarrow$ $\lambda \neq (n+1)\alpha$, $\alpha^{n+1}=1$.

$d(\Delta) = n+1$. $h(0) = h(1) = \cdots = h(n) = 1$.

$Hp(\Delta)$.



$\sum h(i) = n+1$.

Thm. ~~Let $p \nmid (n+1)$~~. $f(\lambda, x)$ is Δ-regular $/\mathbb{F}_q$. $\Rightarrow$

1) $L(x, f, T)^{(-1)^n} = \prod_{i=0}^{n} (1 - \alpha_i(\lambda) T)$.

2) $\alpha_0(\lambda) = 1$. $|\alpha_i(\lambda)| = q^{\frac{n+1}{2}}$ $(1 \leq i \leq n)$.

3) Generically ordinary $\forall p \nmid (n+1)$.

For all but finitely many λ, $\Rightarrow$

$\mathrm{ord}_q(\alpha_i(\lambda)) = i$.

21). slope mirror symmetry.

Δ. M_λ : $X_1^5 + \cdots + X_5^5 - \lambda X_1 \cdots X_5 = 0 \quad / \mathbb{F}_q$

smooth proj.

Δ^*. W_λ mirror smooth proj $/ \mathbb{F}_q$.

$$Z(M_\lambda, T) = \frac{P(M_\lambda, T)}{(1-T)(1-qT)(1-q^2T)(1-q^3T)}$$

$\deg P(M_\lambda, T) = 204$, pure of weight 3.

$$Z(W_\lambda, T) = \frac{P(W_\lambda, T)}{(1-T)(1-qT)^{101}(1-q^2T)^{101}(1-q^3T)}$$

$\deg P(W_\lambda, T) = 4$.

$HP(\Delta)$:

$\|$

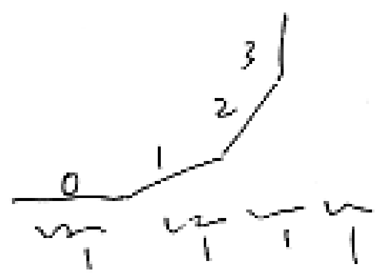
$HP(M_\lambda)$

$= HP\left(\text{denomin of } \cancel{HP(W_\lambda)}\ Z(W_\lambda, T)\right).$

$HP(\Delta^*)$

$\|$

$HP(W_\lambda)$



$= HP\left(\text{denom of } Z(M_\lambda, T)\right)$

$HP(W_\lambda, T)$ and $HP(M_\lambda, T)$ switch under mirror.

Generic slope sym.:

1) $GNP(P(M_\lambda, T)) = NP\left((1-T)(1-\xi T)^{101}(1-\xi^2 T)^{101}(1-\xi^3 T)\right)$

$= HP(M_\lambda)$.

2) $GNP(P(W_\lambda, T)) = NP\left((1-T)(1-\xi T)(1-\xi^2 T)(1-\xi^3 T)\right)$

$= HP(W_\lambda)$.

Thm. 2) is true for all p. (Δ^* is Fano)

1) is true if $p \equiv 1 \pmod 5$.

?: 1) is true for all p ?

22). p-adic analytic formula for zeta.

Let $f(\bar\lambda, x) \in M_p(\Delta)(\bar{\mathbb{F}}_q)$. $q = p^a$.

$\Rightarrow Z(U_f, T) = * \cdot P(f(\bar\lambda, x), T)^{(-1)^n}$.

$P(f(\bar\lambda, x), T) \in 1 + T\mathbb{Z}[T]$, of deg $d(\Delta)-1$.

$$\begin{aligned} &P(f(\bar\lambda,x), T) \\ &\quad \| \\ &\det(I - F(\hat\lambda)T \mid H_0(K_\cdot)). \end{aligned}$$

Thm. 1) Zariski locally on $M_p(\Delta)$, $\Rightarrow$

$$F(\bar\lambda) = A(\lambda^{p^{a-1}}) \cdots A(\lambda^p) A(\lambda), \quad \lambda = \mathrm{Teich}(\bar\lambda).$$

$A(\lambda)$ is a p-adic analytic matrix $/\mathbb{Z}_p$.

2) $p > 2$. One can take $A(\lambda) = C(\lambda^p)^{-1} A(0) C(\lambda)$, 0 is a regular pt.

$C(\lambda)$ = fund sol. matrix of Picard-Fuchs.

Thm. 1) Zariski locally.

$$P_k(\bar\lambda, T) = \det(I - F_k(\bar\lambda) q^k T).$$

$$F_k(\bar\lambda) = A_k(\lambda^{p^{a-1}}) \cdots A_k(\lambda^p) A_k(\lambda).$$

$\bar\lambda \in \bar{\mathbb{F}}_{p^a}$.

$\lambda = \mathrm{Teich}(\bar\lambda)$.

$$A_k(\lambda) \in GL_{h(k+1) \times h(k+1)}(\qquad).$$

24). p-adic rep's and unit root L-function

Let Δ be ordinary at p.

$A_k(\lambda)$ is the Frob matrix of a p-adic

Galois rep.

$$\rho_k: \quad \pi_1^{\text{arith}}\left(H_p(\Delta)/\mathbb{F}_p\right) \longrightarrow GL_{R(k+1)}(\mathbb{Z}_p)$$

$$\rho_k(\mathrm{Frob}_{\bar\lambda}) = F_k(\bar\lambda) = A_k(\lambda^{p^{d-1}}) \cdots A_k(\lambda)$$

$$(\bar\lambda \in \mathbb{F}_{p^d}).$$

$$L(\rho_k, T) = \prod_{\substack{\bar\lambda \in H_p(\Delta) \\ \text{closed pt}}} \frac{1}{\det(I - \rho_k(\mathrm{Frob}_{\bar\lambda}) T^{\deg(\bar\lambda)})}$$

$$\in 1 + T(\mathbb{Z}_p^{A(\bar{Q})})[[T]]$$

25). Dwork's Conj.

Thm. $L(\rho_R, T)$ is p-adic mero in T.

$$\| \quad \frac{\prod_{i=1}^{\infty}(1-\alpha_i T)}{\prod_{j=1}^{\infty}(1-\beta_j T)}, \quad \alpha_i \to 0, \quad \beta_j \to 0.$$

Question: $\mathrm{ord}_q(\alpha_i) = ?$ $\quad \mathrm{ord}_q(\beta_j) = ?$