II. Newton polygon.

$$L(x_0 f, T)^{(-1)^n} = \prod_{i=1}^{d(\Delta)} (1-\alpha_i T), \quad \alpha_i \in \overline{Q_p}$$

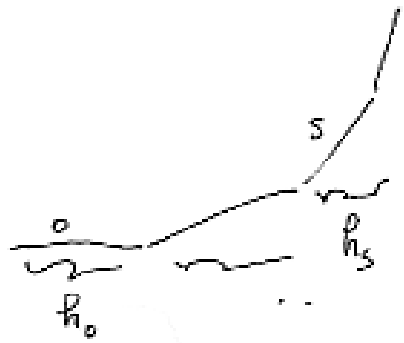
$$|\alpha_i|_q = q^{-s_i}, \qquad s_i = \mathrm{ord}_q(\alpha_i).$$

$$s_i \in Q \cap [0, n+1].$$

<u>Def</u>. $h_s = \#\{1 \le i \le d(\Delta) \mid s_i = s\}$. $s \in Q \cap [0, n+1]$

q-adic NP.

$NP(f)$:



$\sum h_s = d(\Delta)$.

<u>Question</u>: $NP(f) = ?$

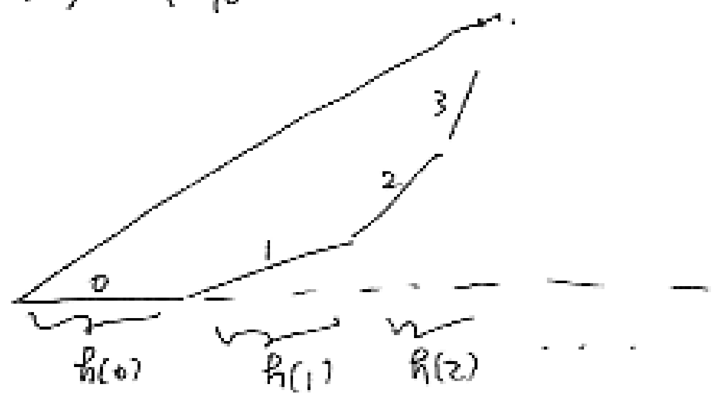
(2). Hodge polygon.

$\Delta \subset \mathbb{R}^n$, n-dim integral convex

$$W(k) = \#(\mathbb{Z}^n \cap k\Delta).$$

$$\sum_{k=0}^{\infty} W(k) T^k = \frac{\sum_{k=0}^{n} h(k) T^k}{(1-T)^{n+1}}.$$

Def. The Hodge polygon of Δ is the polygon



$\sum h(k) = d(\Delta)$

Thm. $NP(f) \geq HP(\Delta)$ with endpts coincide.

14. Geometric variation.

$$M_p(\Delta) = \{ f \in \bar{\mathbb{F}}_p[x_1^{\pm 1}, \ldots, x_n^{\pm 1}] \mid \Delta(f) = \Delta, \quad f \ \Delta\text{-regular} \}.$$

<u>Question</u>: when $M_p(\Delta) \neq \phi$?

<u>Ex</u>. $p > d(\Delta)$. $\Rightarrow$ $M_p(\Delta) \neq \phi$:

$f \in M_p(\Delta)(\bar{\mathbb{F}}_p)$ $\Rightarrow$ $f \in M_p(\Delta)(\mathbb{F}_q)$ for some $\mathbb{F}_q/\mathbb{F}_p$.

$\Rightarrow$ q-adic $NP(f)$ defined.

indep of $\mathbb{F}_q$.

The relative GK $H_0(K_\cdot)$ is "locally free" over $M_p(\Delta)$.

$\Rightarrow$ an F-crystal over $M_p(\Delta)$.

By G–K.

Thm. 1) $\{ NP(f) \mid f \in M_p(\Delta)(\bar{\mathbb{F}}_p) \}$

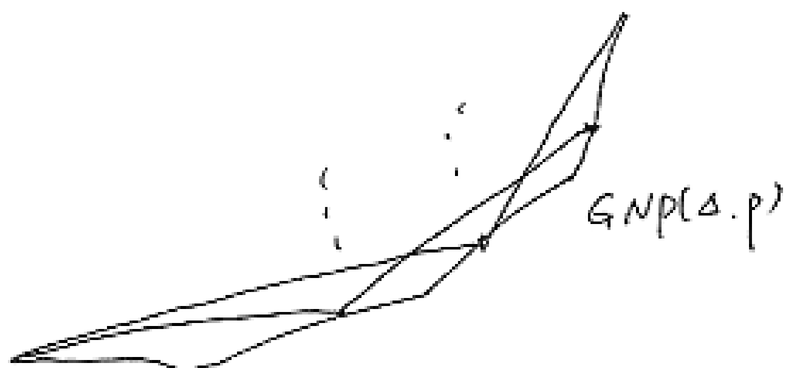
$\exists$ a unique initial element, $GNP(\Delta, p)$

wrt the partial ordering "$\geq$".

2) $\exists$ $U_p(\Delta) \hookrightarrow M_p(\Delta)$

open dense

s.t. $NP(f) = GNP(\Delta, p) \iff f \in U_p(\Delta)$.



Newton straitification of $M_p(\Delta)$.

$$NP(f) \geq GNP(\Delta, p) \geq HP(\Delta)$$

$NP(f) = GNP(\Delta, p)$ generically; $GNP(\Delta, p) \overset{?}{=} HP(\Delta)$

Def. If $GNP(\Delta, p) = HP(\Delta)$,

$\Rightarrow$ Δ is ordinary at p

(or p is ordinary for Δ).

Question: which p is ordinary for Δ?

(5) ordinary primes.

Conj (AS). Δ is ordinary for all $p >> 0$.

Thm A. 1) $\exists$ $D(\Delta) > 0$ s.t. if $p \equiv 1 \pmod{D(\Delta)}$

$\Rightarrow$ p is ordinary for Δ.

2) If $n \leq 3$ $\Rightarrow$ ($D(\Delta) = 1$)

p is ordinary $\forall p > d(\Delta)$.

3) If $n \geq 4$. $\exists$ n-dim Δ s.t

Δ is NOT ordinary for all

p in a residue class of some $D(\Delta)$.

16. Local theory.

Lemma: If Δ is indecomp (no lattice pts $\neq$ vertices) and $p \equiv 1 \ (d(\Delta)) \Rightarrow \Delta$ is ordin at p.

pf. Gauss sums, + Stickelberger.

$(d(\Delta) = n! \, \mathrm{Vol}(\Delta).)$

Cor 1. If $n \leq 2$ and Δ indecomp

$\Rightarrow d(\Delta) = 1 \Rightarrow \Delta$ is ordin $\forall$ p.

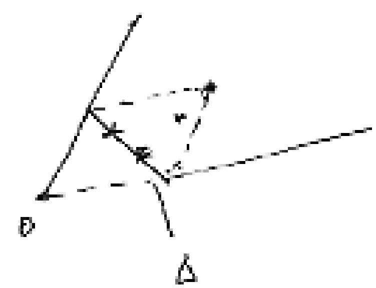
Lemma 2: Let $\Delta = \langle V_0, V_1, \ldots V_n \rangle$ be indecomp.

$p \nmid d(\Delta)$. Then

p is ordinary for Δ

$\Longleftrightarrow$ degree of pts in $\mathbb{Z}^{n+1} / \langle (1, V_0), \ldots (1, V_n) \rangle \mathbb{Z}^{n+1}$

is stable under mult by p.



$\deg(u) = \deg(\overline{pu})$.

Cor 2. If $n \geq 3$, Δ indecomp

$\Rightarrow$ degree is p-stable

$\Rightarrow$ p is ordinary.

Cor 3. If $n=4$, Δ indecomp. $\underset{\text{vertex}}{0} \in \Delta$

$\Rightarrow$ Δ is ordinary for $p \nmid d(\Delta)$.

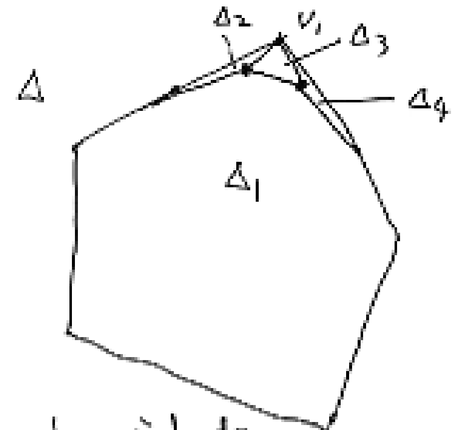
17). Global collapsing decomp.

Thm. $\Delta = \bigcup_{i=1}^{k} \Delta_i$ be a complete collapsing decomp.

If p is ordinary for each Δ_i.

$\Rightarrow$ p is ordinary for Δ.

$(p > d(\Delta))$.



Continue by induction ...

$\Rightarrow$ decompose Δ into indecomp pieces

<u>Cor</u>. Thm A 1) follows.

Thm A 2) follows $(n \le 3)$