1) Periods

CY property in $c_1 = 0 \iff \exists\, \Omega$.

$$\Omega = \frac{1}{2\pi i} \oint \frac{\mu}{P}$$

$$= \epsilon_{ybke} \frac{X^i dX^j dX^k dX^l}{\frac{\partial P}{\partial X^m}} \quad \text{(no sum)}$$

$$\varpi_0 = \frac{(-5\psi)}{(2\pi i)^3} \int_{\substack{|X^j| = \epsilon \\ j = 2,3,4}} \frac{X^1 dX^2 dX^3 dX^4}{\frac{\partial P}{\partial X^5}} = \frac{(-5\psi)}{(2\pi i)^4} \int \frac{X^1 dX^2 \ldots dX^5}{P}$$

$$= \frac{(-5\psi)}{(2\pi i)^5} \int_{|X^i| = \epsilon} \frac{dX^1 \ldots dX^5}{P}$$

$$= \frac{(-5\psi)}{(2\pi i)^5} \int \frac{d^5 x}{(-5\psi \Pi x)\left[1 - \frac{\sum (x^i)^5}{5\psi \Pi x}\right]}$$

$$= \frac{1}{(2\pi i)^5} \int \frac{d^5 x}{(\Pi X)} \sum_{r=0}^{\infty} \frac{\left(\sum x_i^5\right)^r}{(5\psi)^r (\Pi x)^r} \qquad (\Pi x)^r \text{ must occur in } \left(\sum x_i^5\right)^r \;\; \Lambda (x_i^5)^{r/5}$$

$$= \sum^{\infty} \frac{(5m)!}{(m!)^5} \frac{1}{(5\psi)^{5m}}$$

$$\varpi_0 = \sum_{m=0} \frac{(5m)!}{(m!)^5} \lambda^{m} \qquad \lambda = \frac{1}{(5\psi)^5}$$

$$\theta = \lambda \frac{d}{d\lambda}$$

Check $\mathcal{L}\varpi_0 = 0 \qquad \mathcal{L} = \theta^4 - 5\lambda \prod_{i=1}^{4} (5\theta + i)$

Trd

$M: \sum x_i^5 - 5\psi \prod x_i = 0$

$G : x_i \to \alpha^{n_i} x_i \qquad \sum n_i \equiv 0 (5) \; \alpha^5 = 1,$

$\mathbb{Z}_5^3$

M can be deformed

$M \to \sum x_i^5 - 5\psi \prod x_i + \sum_\nu c_\nu x^\nu$

x^ν 100 quintic deformations

41000	20	x^ν	each x^ν transfs in a rep of G.
32000	20		
31100	30		
22100	30		
21110	$5.4 = 20$		$x^i \to M^i_j x^j$ 25

$\int_{\Gamma_J} \Omega$ ~~$\mathfrak{A}$~~

$$h^{pq} = \begin{matrix} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & 1 & & 0 & \\ 1 & & 101 & & 101 & & 1 \\ & 0 & & 1 & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{matrix}$$

More generally

$1 \qquad 101 \qquad 101 \qquad 1$

~~Q...~~

$$1 \longrightarrow Q \longrightarrow Q^2 \longrightarrow Q^3 \qquad \mathcal{L}$$

$$x^\nu \longrightarrow Q x^\nu \qquad \mathcal{L}_\nu$$

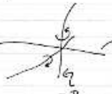
$$\frac{\partial}{\delta \partial^\psi} \frac{1}{P} = + \frac{\varepsilon Q}{P^2}$$

linear

Ideal $\quad x^i \to x^i + \varepsilon^i(x)$

$$P(x) \to P(x) + \varepsilon^i \frac{\partial P}{\partial x^i} \qquad \mathcal{J} = \left(\frac{\partial P}{\partial x^i}\right)$$

$$\int_\Gamma \frac{d^5x}{P} \qquad \int_\Gamma d^5x \, \frac{x^{m}}{P^{1+\frac{1}{5}\deg(m)}}$$



$$\int \frac{d^5x}{P} \qquad \int d^5x \, \frac{Q}{P^2} \qquad \int d^5x \, \frac{Q^2}{P^3} \qquad \int d^5x \, \frac{Q^3}{P^4}$$

all satisfy same eqn.

$$\mathcal{L} \varpi = 0$$

$$\int d^5x \, \frac{x^v}{P^2} \qquad \int d^5x \, \frac{x^v Q}{P^3}$$

sat a 2nd order eqn

$$\mathcal{L}_v \varpi_v = 0$$

The soln with no logs is

$$\varpi_v = \sum^{\infty} \lambda^{m} \underline{(5m)!} = {}_2F_1(a_v, b_v; c_v; \psi^{-5})$$

$$\mathcal{L}_\nu = \theta(\theta - 1 + c_\nu) - 5^5\lambda(\theta + a_\nu)(\theta + b_\nu)$$

ν	γ_ν	$\{a_\nu, b_\nu, c_\nu\}$
41000	20	$\{2/5, 3/5, 1\}$
32000	20	$\{1/5, 4/5, 1\}$
31100	30	$\{1/5, 3/5, 4/5\}$
22100	30	$\{1/5, 2/5, 3/5\}$

$c_\nu = a_\nu + b_\nu$

Method of Frobenius.

$$\mathcal{L}\varpi(\lambda, \varepsilon) = \varepsilon^4 \lambda^\varepsilon$$

$$\varpi(\lambda, \varepsilon) = \sum_{0}^{\infty} A_n(\varepsilon)\, \lambda^{n+\varepsilon}$$

$$\theta\lambda^m = m\lambda^m$$

$$\mathcal{L} = \theta^4 - 5\lambda \prod_{i=1}^{4}(5\theta + i)$$

$\underline{n+\varepsilon)}$ $n \neq 0$ $\quad (n+\varepsilon)^4 A_n - 5\prod_{1}^{4}\left(5(n+\varepsilon) - 5 + i\right) A_{n-1} = 0$

$$A_n = \frac{5\prod_{i=1}^{4}(5n - 5 + i + 5\varepsilon)\, A_{n-1}}{(n+\varepsilon)^4}$$

$$A_n = \frac{\Gamma(5m + 1 + 5\varepsilon)}{\Gamma^5(m + 1 + \varepsilon)}\;\frac{\Gamma^5(\varepsilon)}{\Gamma(5\varepsilon + 1)}$$

$$A_0(\varepsilon) = 1$$

$$\mathcal{L}\varpi(\lambda,\varepsilon) = \theta^4\lambda^\varepsilon = \varepsilon^4\lambda^\varepsilon$$

$\varpi(\lambda,0) \ \cdots \ \frac{\partial^3}{\partial\varepsilon^3}\varpi(\lambda,0)$ solve eqn.

$$\varpi_0 = f_0$$

$$\varpi_1 = f_0\log\lambda + f_1$$

$$\varpi_2 = f_0\log^2\lambda + 2f_1\log\lambda + f_2$$

$$\varpi_3 = f_0\log^3\lambda + 3f_1\log^2\lambda + 3f_2\log\lambda + f_3$$

$$\nu = \nu_0 + \nu_1 p + \nu_2 p^2 + \nu_3 p^3 + \nu_4 p^4$$

$$\varpi(\lambda,\varepsilon) = \sum A_n(\varepsilon)\lambda^{n+\varepsilon}$$

$$A_n(\varepsilon) = \frac{\Gamma(5n+5\varepsilon+1)}{\Gamma^5(n+\varepsilon+1)}\,\frac{\Gamma^5(\varepsilon+1)}{\Gamma(5\varepsilon+1)}$$

$$A_0(\varepsilon) = 1$$

$$\varpi(\lambda,\varepsilon) \to h(\varepsilon) \sum_n{}' A_n(\varepsilon)\, \lambda$$

$$h(0)=1 \qquad h(\varepsilon) = 1 + h_1\varepsilon + \frac{1}{2}h_2\varepsilon^2 + \frac{1}{3!}h_3\varepsilon^3 + \cdots$$

$$\varpi = \varpi_0 + \varpi_1\varepsilon + \frac{1}{2}\varpi_2\varepsilon^2 + \frac{1}{3!}\varpi_3\varepsilon^3$$

$$\begin{aligned} h\varpi &= \left(\varpi_0 + \varpi_1\varepsilon + \frac{1}{2}\varpi_2\varepsilon^2 + \frac{1}{3!}\varpi_3\varepsilon^3\right)\left(1 + h_1\varepsilon + \frac{1}{2}h_2\varepsilon^2 + \frac{1}{3!}h_3\varepsilon^3\right) \\ &= \varpi_0 + (\varpi_1 + h_1\varpi_0)\varepsilon + \frac{1}{2!}\varepsilon^2(\varpi_2 + 2h_1\varpi_1 + h_2\varpi_0) \\ &\quad + \frac{\varepsilon^3}{3!}(\varpi_3 + 3h_1\varpi_2 + 3h_2\varpi_1 + h_3\varpi_0) \end{aligned}$$

$$\begin{pmatrix} \varpi_0 \\ \varpi_1 \\ \varpi_2 \\ \varpi_3 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 0 \\ h_1 & 1 & 0 & 0 \\ h_2 & 2h_1 & 1 & 0 \\ h_3 & 3h_2 & 3h_1 & 1 \end{pmatrix} \begin{pmatrix} \varpi_0 \\ \varpi_1 \\ \varpi_2 \\ \varpi_3 \end{pmatrix}$$

$$\nu_\lambda = \sum_n{}' \left(1 - p^{(n)(p-1)}\right) + O(p)$$

$$= {}^{[n\beta]}\!\varpi_0(\lambda)$$

$$\nu_\lambda = \sum_n \left(1 - p^{\,p(p-1)}\right) + O(p^2)$$

$$p^{(p-1)} = 1 + O(p) \qquad p^{p(p-1)} = 1 + O(p^2)$$

$$\nu_\lambda = {}^{[2\psi]}f_0(\lambda^p) + p\,{}^{[2\psi]}f_1'(\lambda^p)$$

$$+ \varepsilon_{\frac{2\psi}{5}}\, p \sum_\nu \frac{\gamma_\nu}{\prod_i (\nu_i k)!}\, {}^{[\psi]}_{2}F_{1}(a_\nu, b_\nu; c_\nu; \psi^{-5}) + O(p^2)$$

To proceed make an ansatz.

$$\cancel{\nu_\lambda = {}^{[3\psi]}f_0(\lambda p^2) +}$$

$$\nu = (1 + Ap^2)\, {}^{[3\psi]}f_0(\lambda^{p^2}) + p(1 + Bp)\, {}^{[\psi]}f_1'(\lambda^{p^2})$$

$$+ Cp^2\, {}^{[3\psi/5]}f_2''(\lambda^{p^2}) + O(p^3)$$

fix A, B, C.

$$\nu = {}^{(\psi)}f_0(\lambda^{p^4}) + \frac{p}{1-p}\, {}^{(\psi)}f_1'(\lambda^{p^4}) + \frac{1}{2!}\left(\frac{p}{1-p}\right)^2 {}^{(\psi)}f_2''(\lambda^{p^4})$$

$$+ \cdots + \frac{1}{4!}\left(\frac{p}{1-p}\right)^4 {}^{(\psi)}f_4''''(\lambda^{p^4})$$

$$+ \cancel{\varepsilon_3} h_3 \left(\frac{p}{1-p}\right)^3 \left\{ {}^{(\psi)}f_0''' + \left(\frac{p}{1-p}\right) {}^{(\psi)}f_1'''' \right\} + O(p^5)$$

$$h(\varepsilon) = 1 + \frac{1}{3!} h_3 \varepsilon^3 + O(\varepsilon^5)$$

$$\frac{a_{rp}}{a_r} = \frac{\Gamma(5rp+1)}{\Gamma^5(rp+1)} \frac{\Gamma^5(r+1)}{\Gamma(5r+1)} = \frac{\Gamma_p(5rp+1)}{\Gamma_p^5(rp+1)} = h(rp)$$

~~[illegible]~~

$h(\varepsilon)$ $\quad \frac{a_{rp}}{a_r} \sim h(rp)$

$$\vartheta_\lambda = \sum_{s=0}^{p-1} \frac{a_{s(1+p+p^2+\cdots p^4)}}{a_{s(1+p+\cdots+p^3)}} \lambda^{sp^4} + \mathcal{O}(p^5)$$

If correct with $h(\varepsilon)$ then.

$$\vartheta = {}^{(p)}f_0(\mathrm{Teich}\,\lambda) + \frac{p}{1-p}\, {}^{(p)}f_1'(\mathrm{Teich}\,\lambda)$$

$$+ \frac{1}{n!}\left(\frac{p}{1-p}\right)^n {}^{(p)}f_n^{(n)}(\mathrm{Teich}\,\lambda) + \cdots$$

exactly

Note occurrence of $f_4, f_5, \ldots$ etc.

$$\vartheta = \sum_{m=0}^{p-1} \beta_m (\mathrm{Teich}\,\lambda)^m$$

$$\beta_m = \lim_{n\to\infty} \frac{a_{m(1+p^2+\cdots+p^{n+1})}}{a_{m(1+p+\cdots+p^n)}}$$

Fun with $\dim = 0$ $\mathbb{P}_1[2]$ $\qquad$ $\mathbb{P}_4[5]$, $\mathbb{P}_3[4]$, $\mathbb{P}_2[3]$, $\mathbb{P}_1$

$$x_1^2 + x_2^2 - 2\psi x_1 x_2 = 0 \qquad \lambda = \frac{1}{(2\psi)^2}$$

$$\mathcal{L} = \theta - 2\lambda(2\theta+1)$$

$$\varpi_0 = f_0 = \frac{1}{\sqrt{1-4\lambda}} = \sum \frac{(2n)!}{(n!)^2} \lambda^n$$

$$\varpi_1 = f_0 \log\lambda + f_1$$

$$f_1 = 2\sum_1^\infty \frac{(2n)!}{(n!)^2} (\sigma_{2n} - \sigma_n) \lambda^n$$

$$\sigma_n = \sum_{k=1}^n \frac{1}{k}$$

Calculate N_λ from f_0 and f_1

$\dim = -1 \qquad \mathbb{P}_0[1] \qquad x \sim \ell x$

$$\underline{P}(x) = x - \psi x = (1-\psi)x = 0$$

$$\lambda = \frac{1}{\psi}$$

$$f_0 = \omega_0 = \sum \frac{n!}{n!}\lambda^n = \sum \lambda^n = \frac{1}{1-\lambda}$$

$$\nu_\lambda = \begin{cases} 1 & \text{if } \lambda \neq 1 \\ p & \text{if } \lambda = 1 \end{cases}$$

$$\nu_\lambda = {}^{[p]}f_0(\lambda) = \sum_{n=0}^{p-1} \lambda^n = \begin{cases} \frac{\lambda^p - 1}{\lambda - 1} = 1 & \lambda \neq 1 \\ p & \lambda = 1 \end{cases}$$

Dwork's character

$$\cancel{\Theta(x)} \qquad \exp(\pi(X - X^p))$$

$$\Theta(x) \qquad E(\pi X) \qquad X = \mathrm{Teich}(x)$$

$$\sum_{y \in \mathbb{F}_p} \Theta(y P(x)) = \begin{cases} p \\ 0 \end{cases}$$

$$\cancel{\Theta(X+Y) = \Theta(X)\Theta(Y)}$$

$$\Theta(x+y) = \Theta(x)\Theta(y)$$

$$\Theta(yP) = \Theta\left(y\left(\sum x_i^5 - 5\psi \prod x_i\right)\right)$$

$$= \Theta\left(-5\psi y \prod x_i\right) \prod_i \Theta\left(y x_i^5\right)$$

$$G_n = \sum_{x \in \mathbb{F}_p^*} \Theta(x) \left(\mathrm{Teich}\, x\right)^n$$

$$g_n(y) = \sum_{x \in \mathbb{F}_p^*} \Theta(yx^5) \left(\mathrm{Teich}\, x\right)^n$$

$$\Theta(x) = \frac{1}{p-1} \sum_{m=0}^{p-2} G_{-m} \left(\mathrm{Teich}\, x\right)^m$$

$$\sum_{x \in \mathbb{F}_p^*} \left(\mathrm{Teich}\, x\right)^n = \begin{cases} 0 & \text{if } p-1 \nmid n \\ p-1 & \text{if } p-1 \mid n \end{cases}$$

$$\nu = 1 + p^4 + \sum_{m=1}^{p-2} \frac{G_m^5}{G_{5m}} \left(\mathrm{Teich}\, \lambda\right)^{-m} \qquad 5 \nmid p-1$$

when $5 \mid p-1$ $\qquad k = (p-1)/5$

$$\to p^4 - (p-1)^5 = \sum \nu \sum^{p-2} (-1)^m \mathrm{Teich}(\lambda)^m G_r \prod^5 G_{\dots}$$

Main themes

Kähler class

Frobenius period

Octic – coords and cohomology ring

fundamental period.

K-class

M $\mathbb{P}_4[5]$

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 1 & & 0 & \\
h^{pq} = 1 & & 101 & & 101 & & 1 \\
 & 0 & & 1 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
$$

H which is Kähler form of $\mathbb{P}^4$

Mirror has $h^{11} = 101$ size of divisors

W

M has 204 periods W has 4 $\varpi_0, \varpi_1, \varpi_2, \varpi_3$

$\underbrace{\varpi_0\ \varpi_1\ \varpi_2\ \varpi_3}$

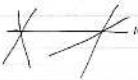
$1\ Q\ Q^2\ Q^3$

$x^\vee,\ Qx^\vee$

x^v correspond (monomial divisor mirror map) to divisors D_v of W

~~Eg~~ $W = \widehat{M/G}$

$\frac{5 \cdot 4}{2} = 10$ faces



~~3/4/4~~ ~~Each~~ $10 \times 3/3 = 10$

C_y $\frac{5 \cdot 4}{2} = 10$

triangulation

$$\zeta_W = \frac{R_1}{(1-T)(1-pT)^{101}(1-pT)^{151}(1-p^2T)}$$

$$= \frac{R_1}{(1-T)(1-pT)(1-p^2T)(1-p^3T)} \times \frac{1}{(1-pT)^{100}(1-p^2T)^{150}}$$

Should perhaps use a separate T for each D_r.

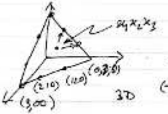
Reflexive polyhedra

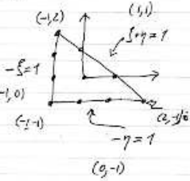
Elliptic curve

$$\sum x_i^3 - 3\psi x_1 x_2 x_3 = 0$$

$x^{\underline{m}}$ $\quad \underline{m} = (m_1, m_2, m_3)$

$m_i \geq 0 \quad m_1 + m_2 + m_3 = 3$





Δ polyhedron of monomials $x^{\underline{m}}$

∇ polyhedron over the fan of the toric variety.

$\mathcal{M} \sim (\Delta, \nabla)$ given (Δ, ∇) can reconstruct family $\ni \mathcal{M}$

$W \sim (\nabla, \Delta)$

$\underline{v}$ is a monomial of $\mathcal{M}$
$\underline{v}$ is a divisor of W.

Frobenius period.

$$\varpi(\lambda, \varepsilon) = \sum_{n=0}^{\infty} A_n(\varepsilon)\, \lambda^{n+\varepsilon}$$

$$\varepsilon^4 = 0,\ \varepsilon^3 \neq 0$$

ε is Kähler form

$\mathcal{M}$ CY hypersurface in $\mathbb{P}_\nabla$

$$\mathcal{P}(x) = \sum_{\substack{m \in \Delta \\ m \neq \underline{1}}} c_m x^{\underline{m}} - c_1 x^{\underline{1}}$$

$$\varpi(\cancel{c}c, D) = \frac{\prod_m{}' \Gamma(D_m+1)}{\Gamma(-D_1+1)} \sum_{\gamma \in V_\nabla} \frac{\Gamma(-\gamma \cdot D_1 - D_1 + 1)}{\prod_m{}' \Gamma(\gamma \cdot D_m + D_m + 1)} \times c^{\gamma + D}$$

$$\prod{}' = \prod_{\substack{m \in \Delta \\ m \neq 1}}$$

D_m toric divisors of $\mathbb{P}_\nabla$ $\leftrightarrow$ $m \in \nabla$

$$D_1 \overset{\text{def}}{=} -[W] = -\sum_m{}' D_m$$

Sum over curves γ in the Mori cone of $\mathbb{P}_\nabla$
$\gamma \cdot D_m$ = intersection numbers.

Can think of γ as vector in dual space to that spanned by D_m

$D = (D_1, D_{\underline{m}})$ ~~vector~~ length $\# m \in \nabla$

$\gamma = (\gamma_1, \gamma_{\underline{m}})$

$$c^{\gamma + D} = \prod_{m \in \Delta} c_m^{\gamma_m + D_m}$$

Apply to mirror quintic.

$$\begin{array}{c} D_0 \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \end{array} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 5 & & & & \\ 1 & & 5 & & & \\ 1 & & & 5 & & \\ 1 & & & & 5 & \\ 1 & & & & & 5 \end{bmatrix}$$

$$\underline{P} = c_1 x_1^5 + c_2 x_2^5 + \cdots + c_5 x_5^5 - c_0 x_1 x_2 x_3 x_4 x_5$$

$$x_i = y_i / c_i^{1/5}$$

$$\underline{P} = \sum y_i^5 - 5\psi\, y_1 y_2 \cdots y_5$$

$$\lambda = \frac{1}{(5\psi)^5} = \frac{c_1 c_2 c_3 c_4 c_5}{c_0^5} = c^k$$

$k = (-5, 1 1 1 1 1)$
generator of Mori cone

$$D_0 = -(D_1 + D_2 + \cdots + D_5)$$

$$D_0 = -5 D_1$$

$$D_0 = -5 D_5$$

$D_1 = D_2 = \ \ = D_5 = H$ say $\quad D_0 = -5H$

$\gamma = nk$

$$\varpi(c, D) = \frac{\Gamma^5(H+1)}{\Gamma(5H+1)} \sum_{n=0}^{\infty} \frac{\Gamma(5n+5H+1)}{\Gamma(n+H+1)^5} \lambda^{n+H}$$

$k \cdot H = 1 \qquad -\gamma \cdot D_0 = -\gamma(-5H) = 5$

$D_0 = -5H$

$\varpi(\lambda, H)$

$\mathbb{P}^{(11222)}[8] \qquad (x_1, x_2, x_3, x_4, x_5) \sim (r x_1, r x_2, r^2 \ldots$

$\sum$ weight = degree.

$$\epsilon_{ijklm} \frac{x^i dx^j dx^k dx^l dx^m}{P}$$

$$P = y_1^8 + y_2^8 + y_3^4 + y_4^4 + y_5^4 - 2\phi y_1^4 y_2^4 - 8\psi y_1 y_2 y_3 y_4 y_5$$

such that P is most general octic.

$W = \widehat{M}/G$

$$\mathcal{P} = c_1 x_1^8 + c_2 x_2^8 + c_3 x_3^4 \cdots + c_5 x_5^4$$
$$+ c_6 x_1^4 x_2^4 - c_0 x_1 x_2 x_3 x_4 x_5$$

Scale coords as before

$$\lambda = \frac{a_3 a_4 a_5 a_6}{c_0^4} \qquad \mu = \frac{c_1 c_2}{c_6^2}$$

$$= -\frac{2\phi}{(8\psi)^4} \qquad = \frac{1}{(2\phi)^2}$$

D_0	1	1	1	1	1	1
D_1	1	8	0	0	0	0
D_2	1	0	8	0	0	0
D_3	1	0	0	4	0	0
D_4	1	0	0	0	4	0
D_5	1	0	0	0	0	4
D_6	1	4	4	0	0	0

$D_0 = -(D_1 + \cdots + D_6)$

$\cancel{D_0 = -8D_1 = -8D_2 = -4D_3}$ $D_1 = D_2 = \cancel{2}\ L$

$\cancel{= -4D_4 = -4D_5}$ $D_3 = D_4 = D_5 = H$

$\cancel{D_0 = -4H}$

$D_0 = -4D_3 = -4D_4 = -4D_5$

$D_3 = D_4 = D_5 = H$

$D_0 = -4H$

$D_0 = -8D_1 - 4D_6$ $D_1 = D_2 = L$ say.

$D_0 = -8D_2 - 4D_6$

$+4H = +8L + 4D_6$ $\underline{D_6 = H - 2L}$

$\cancel{-4H = -8L - 4H + H - 2L}$

$-4H = -[\overset{3}{\cancel{8}}H + \cancel{2L} + H - \cancel{2L}]$ ✓

$k = (-4, 001111)$; $\ell = (011000 - 2)$

$\gamma = nh + m\ell$

$H^3 = 8$ $L^2 = 0$ $H^2L = 4$

$$
\begin{array}{lccccc}
 & & & 1 & & \\
M.\ K & & 0 & & 0 & \\
 & 1 & & 20 & & 1 \\
W & & 0 & & 0 & \\
 & & & 1 & &
\end{array}
$$

H^2

$$Z(M,T) = \frac{1}{(1-T)(1-pT)(1-p^2T)\cdot \underbrace{P(M,T)}_{\deg = 19}}.$$

$$Z(W,T) = \frac{1}{(1-T)(1-pT)(1-p^2T)\, P(W,T)}. \qquad \deg = 19$$

$\underline{\dim N = 2h^{21}+2}$

$\dim D = 2h^{11}+2$

$h^{11} \leftrightarrow h^{31}$

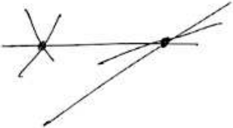
$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 0 & & 0 & & & \\
 & & 0 & & h^{11} & & 0 & & \\
 & 0 & & h^{21} & & h^{21} & & 0 & \\
1 & & h^{31} & & h^{22} & & h^{31} & & 1 \\
 & 0 & & h^{21} & & h^{21} & & 0 & \\
 & & 0 & & h^{11} & & 0 & &
\end{array}
$$

4-fold.

$\underline{\mathbb{P}^4[5]}$

$$h^{pq} = \begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & 1 & & 0 & \\ 1 & & 101 & & 101 & & 1 \\ & 0 & & 1 & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array}$$

<u>Kähler class</u>



$$W = \hat{M}/G$$

W has $h^{11} = h^{21}(M) = 101$

$$\zeta_W = \frac{R_0}{(1-T)(1-pT)^{101}(1-p^2T)^{101}(1-p^3T)}$$

$$= \frac{R_0}{(1-T)(1-pT)(1-p^2T)(1-p^3T)} \times \frac{1}{(1-pT)^{100}(1-p^2T)^{100}}$$

$J = i g_{\mu\bar{\nu}} dx^\mu dx^{\bar{\nu}}$ $\qquad H^{(1,1)} = H^2$

$J = \sum t^i e_i$

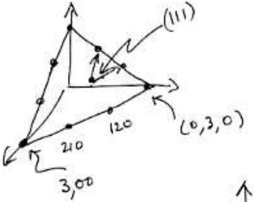
$$\varpi(\lambda, \varepsilon) = \sum_{n=0}^{\infty} A_n(\varepsilon)\, \lambda^{n+\varepsilon} = \sum_{k=0}^{3} \frac{1}{k!} \varpi_k(\lambda)$$

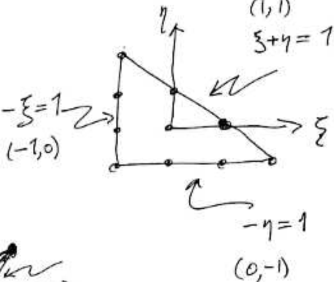
$\mathcal{L}\varpi = 0 \quad \varpi_0, \varpi_1, \varpi_2, \varpi_3 \ , \ \varepsilon^4 = 0,\ \varepsilon^3 \neq 0$

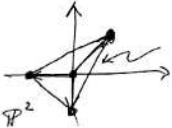
$H^4 = 0$

$\mathbb{P}^2[3]$ $\qquad \sum_{i=1}^{3} x_i^3 - 3\psi x_1 x_2 x_3 = 0 .$

$x^{\underline{m}} \quad \underline{m} = (m_1, m_2, m_3)\ , \ m_i \geq 0, \ m_1+m_2+m_3 = 3$







Δ = Newton poly

∇ = poly over the fan of $\mathbb{P}^2$

$$x^{\underline{m}}$$

$$M \simeq (\Delta, \nabla) \quad \text{fan of } \mathbb{P}_\nabla$$

monomials

toric data constructs a family of manifolds $\ni M$.

$$W \simeq (\nabla, \Delta)$$

$\underline{m}$ is a divisor of the toric variety in which W is a hypersurface.

Apply to mirror quintic.

$$\begin{array}{cc} D_0 & \\ D_1 & \\ D_2 & \\ D_3 & \\ D_4 & \\ D_5 & \end{array}\left[\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 5 & 0 & 0 & 0 & 0 \\ 1 & 0 & 5 & 0 & 0 & 0 \\ 1 & 0 & 0 & 5 & 0 & 0 \\ 1 & 0 & 0 & 0 & 5 & 0 \\ 1 & 0 & 0 & 0 & 0 & 5 \end{array}\right]$$

D_m

$D_0 = -5D_1$

$D_0 = -5D_2$

$\vdots$

$D_0 = -5D_5$

$D_1 = D_2 = \cdots = D_5 = H$

$D_0 = -5H.$

$D = (D_0, D_m)$ $\qquad D_0 = -\sum' D_m$

$\gamma = (\gamma_0, \gamma_m)$ $\qquad \gamma_0 = -\sum' \gamma_m$

$$\underline{P}(x) = c_1 x_1^5 + c_2 x_2^5 + \cdots + c_5 x_5^5 - c_0 x_1 x_2 x_3 x_4 x_5.$$

$$c_i^{-1/5}\, y_i = x_i$$

$$\underline{P} = y_1^5 + y_2^5 + \cdots + y_5^5 - 5\psi\, y_1 y_2 y_3 y_4 y_5.$$

$$5\psi = \frac{c_0}{(c_1 c_2 c_3 c_4 c_5)^{1/5}}$$

$$\sum \frac{(5m)!}{(m!)^5} \lambda^m =$$

$$\lambda = \frac{1}{(5\psi)^5} = \frac{c_1 c_2 \cdots c_5}{c_0^5} = c^k$$

$k = (-5, 1, 1, 1, 1, 1)$

generates Mori cone.

$\gamma = nk \quad , \quad k \cdot H = 1 \quad -\gamma \cdot D_0 = \gamma \cdot (-5H) = 5n$

$$\varpi(\lambda, H) = \frac{\Gamma^5(H+1)}{\Gamma(5H+1)} \sum_{n=0}^{\infty} \frac{\Gamma(5n+5H+1)}{\Gamma^5(n+H+1)} \lambda^{n+H}$$

For the quintic. $\quad R_1 = 1 + aT + bpT^2 + ap^3T^3 + p^6T^4$

$$\zeta_M(T,\psi) = \frac{R_1(T,\psi)\, R_A(pT,\psi)^{20}\, R_B(pT,\psi)^{30}}{(1-T)(1-pT)(1-p^2T)(1-p^3T)}$$

$$\zeta_W(T,\psi) = \frac{R_1(T,\psi)}{(1-T)(1-pT)^{101}(1-p^2T)^{101}(1-p^3T)}$$

$$h^{pq} = \begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & 1 & & 0 & \\ 1 & & 101 & & 101 & & 1 \\ & 0 & & 1 & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array}$$

Expand 5-adically.

$$R_1(T,\psi) = (1-T)(1-pT)(1-p^2T)(1-p^3T) + O(5^2)$$

$$R_A^{20} R_B^{30} = (1-pT)^{100}(1-p^2T)^{100} + O(5^2)$$

$$\zeta_M = \frac{1}{\zeta_W} + O(5^2)$$

$$\frac{y_{ttt}}{5} = 1 + \frac{1}{5}\sum_{k=1}^{\infty} \frac{k^3 n_k q^k}{1-q^k} = 1 + O(5^2).$$

$$5^3 \mid k^3 n_k.$$

$\underline{\psi^5 = 1}$.

125 nodes 125 nodes

$$\zeta(T,\psi^5=1) = \frac{(1-\varepsilon pT)(1-a_p T + p^3T^2)\left(1-(pT)^\rho\right)^{100/\rho}}{(1-T)(1-pT)(1-p^2T)(1-p^3T)\underline{\left[1-(p^2T)^\rho\right]^{24/\rho}}}$$

$\varepsilon = \left(\frac{p}{5}\right) = \pm 1$ $\rho = 1$

a_p is the p'th coeff. in the q-expansion of the wt 4 modular form for $\Gamma_0(25)$.

$\mathcal{J} = \left(\frac{\partial P}{\partial x_4}\right)$ $\frac{\partial P}{\partial x_4} = 5x_4^4 - 5\psi x_1x_2x_3x_5$

x^ν Q_1 + 100 others.

$x_1x_2^2x_3^3x_4^4 \simeq \psi\, x_1^2x_2^3x_3^4x_5 \simeq \dots \simeq \psi^5 x_1x_2^2x_3^3x_4^4$

$F(a_v, b_v, c_v; \psi^{-5})$ $\qquad a+b=c$

$$\int dx\, x^{-\alpha/5}(1-x)^{-\beta/5}\left(1-\frac{x}{\psi^5}\right)^{-(1-\beta/5)} = \int \frac{dx}{y}.$$

$$\boxed{y^5 = x^{\alpha}(1-x)^{\beta}\left(1-\frac{x}{\psi^5}\right)^{5-\beta}} \qquad \mathcal{E}_{\alpha\beta}.$$

	V	(a,b,c)	$\alpha\ \beta$
A	41000	$(2/5\ 3/5\ 1)$	$(2,3)$
A	32000	$(1/5\ 4/5\ 1)$	$(1,4)$
B	31100		
B	22100		

$\alpha = 5(1-b)$

$\beta = 5(1-a)$

$$Z_A(u) = \frac{R_A(u)^2}{(1-u)(1-pu)}$$

$\cancel{N = \Sigma}$ $\quad N = N_0 + \sum_v{}' N_v$

$\qquad\qquad\qquad \varpi_0 \quad \varpi_v$

$$\zeta = \zeta_0 \prod_v \zeta_v \;\rightarrow\; \frac{R_0\, R_A(p^\rho T^\rho, \psi)^{20/\rho}\, R_B(p^\rho T^\rho, \psi)^{30/\rho}}{(1-T)(1-pT)(1-p^2T)(1-p^3T)}$$

$5 \mid q-1$

$5 \mid p^\rho - 1 \qquad \rho = 1, 2 \text{ or } 4$

v

$\left.\begin{array}{l} 41000 \\ 32000 \end{array}\right\rangle \; \zeta_{41000}\, \zeta_{32000} = R_A(p^\rho T^\rho, \psi)^{1/\rho}$

$\left.\begin{array}{l} 31100 \\ 22100 \end{array}\right\rangle \; \zeta_{31100}\, \zeta_{22100} = R_B(p^\rho T^\rho, \psi)^{1/\rho}$

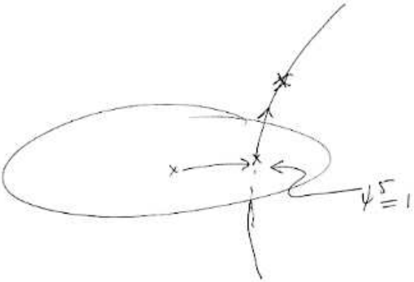
$$\zeta(T,\psi) = \exp\left(\sum_{k=1}^{\infty} \frac{N_k(\psi)\, T^k}{k}\right)$$

$$\zeta = \frac{\text{Numerator of degree } b^3 = 2h^{21}+2 \ \text{ depends on params } \psi}{\begin{array}{c}\text{Denominator of degree } 2h^{11}+2 \\ (1-T)(1-pT)(1-p^2T)(1-p^3T)\end{array}}$$

← does not depend on the Kähler class params.

$$\zeta_M \overset{?}{=} \frac{1}{\zeta_W}$$

$$h^{pq} = \begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{11} & & 0 & \\ 1 & & h^{21} & & h^{21} & & 1 \\ & 0 & & h^{11} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array}$$



$h^{21} = \cancel{25}\,0$

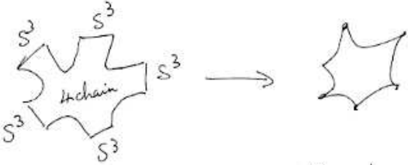
$h^{11} = 25$

125 nodes. $S^3 \longrightarrow 0$.

101 params.

---

24 relations



24 relations

24 cycles.

Octics Shabnam Kadir

$\mathbb{P}^4_{(11222)}[8]$ $(x_1,x_2,x_3,x_4,x_5) \sim (\lambda x_1, \lambda x_2, \lambda^2 x_3, \lambda^2 x_4, \lambda^2 x_5)$

$$\overset{\lambda=-1}{\sim} (-x_1,-x_2,x_3,x_4,x_5)$$

FPS $(0,0,x_3,x_4,x_5)$

$$\mathcal{M}: \quad P = x_1^8 + x_2^8 + x_3^4 + x_4^4 + x_5^4 - 2\phi x_1^4 x_2^4 - 8\psi x_1 x_2 x_3 x_4 x_5$$

$$G \quad (x_1,x_2,x_3,x_4,x_5) \longrightarrow (\alpha^{n_1}x_1, \ldots, \alpha^{n_5}x_5)$$

$$\alpha^8 = 1 \qquad a = n_1+n_2+2n_3+2n_4+2n_5, \quad a \equiv 0 \ (8)$$

$G \cong \mathbb{Z}_4^3$ h^{11}

$$\mathcal{W} = \widehat{\mathcal{M}/G}$$

$$h^{11}(M) = \underline{\underline{2}} \qquad h^{21}(M) = 86 = 83+3$$

$$(x_1, \dots, x_5, \psi, \phi) \longrightarrow (\alpha^{n_1} x_1, \dots, \alpha^{n_5} x_5, \psi \alpha^{-a}, \phi \alpha^{-4a})$$

$$\psi^8, \ \psi^4 \phi, \ \phi^2.$$

$$\Delta, \nabla$$

$$h^{21} = \overbrace{\mathrm{pts}(\Delta) - \sum_{\substack{\operatorname{codim}\theta = 1 \\ \theta \in \Delta}} \mathrm{int}(\theta)}^{83} + \overset{3}{\sum_{\substack{\operatorname{codim}\theta = 2 \\ \theta \in \Delta.}} \mathrm{int}(\theta)\,\mathrm{int}(\theta^*)} - 5$$

$$h^{11} = \mathrm{pts}(\nabla) - \sum_{\substack{\operatorname{codim}\theta^* = 1 \\ \theta^* \in \nabla}} \mathrm{int}\,\theta^* + \sum \mathrm{int}(\theta^*)\,\mathrm{int}(\theta) - 5$$

$$P = x_1^8 + x_2^8 + x_3^4 + x_4^4 + x_5^4 - 2\phi x_1^4 x_2^4 - 8\psi x_1 \cdots x_5.$$

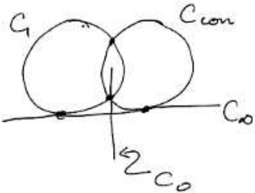
ϕ, ψ

C_{con} (1) $(\phi + 8\psi^4)^2 - 1 = 0$ conifold locus: $\mathcal{M}$ has nodes.

C_1 (2) $\phi^2 = 1$, $\mathcal{M}$ has 4 rather singular pts.

C_∞ (3) $\phi, \psi \to \infty$ $\mathcal{M}$ singular

C_0 (4) $\psi = 0$ orbifold $\psi \simeq \alpha\psi$



$\phi = \pm 1$

$$P = (x_1^4 \pm x_2^4)^2 + x_3^4 + x_4^4 + x_5^4 - 8\phi x_1 \cdots x_5$$

$$\sum_{y \in \mathbb{F}_p} \omega(yP) = \delta(P(x))$$

$$\omega(yP) = \omega(-84\, y\, x_1 x_2 \cdots x_5)\, \omega(-20\, y\, x_1^4 x_2^4)$$

$$\omega(y x_1^8)\, \omega(y x_2^8)\, \omega(y x_3^4)\, \omega(y x_4^4)\, \omega(y x_5^4)$$

$$\omega(\xi) = \frac{1}{p-1} \sum_{m=0}^{p-2} G_{-m}\, \mathrm{Teich}(\xi)^m$$

double sums.

$$\mathcal{M}: \quad \zeta(T,\psi,\phi) = \frac{\overset{\text{sextic}}{R_1} \prod_v R_v^{\gamma_v}}{(1-T)(1-pT)^2(1-p^2T)^2(1-p^3T^{\cancel{2}})}$$

$$R_1\left(\frac{1}{p^3T}\right) = \frac{1}{p^9T^6} R_1(T)$$

$$W: \quad \zeta_W = \frac{R_1}{(1-T)(1-pT)^{83}(1-p^2T)^{83}\left(1-\left(\frac{\phi^2-1}{p}\right)pT\right)^3\left(1-\left(\frac{\phi^2-1}{p}\right)p^2T\right)^3(1-p^3}$$

$$\underline{P} = c_1 y_1^8 + c_2 y_2^8 + c_3 y_3^4 + \cdots + c_6 y_1^4 y_2^4 + c_7 y_1 y_2 y_3 y_4 y_5$$

$$\begin{array}{lcccccc}
D_0 & 1 & 1 & 1 & 1 & 1 & 1 \\
D_1 & 1 & 8 & 0 & 0 & 0 & 0 \\
D_2 & 1 & 0 & 8 & 0 & 0 & 0 \\
D_3 & 1 & 0 & 0 & 4 & 0 & 0 \\
D_4 & 1 & 0 & 0 & 0 & 4 & 0 \\
D_5 & 1 & 0 & 0 & 0 & 0 & 4 \\
D_6 & 1 & 4 & 4 & 0 & 0 & 0
\end{array}$$

$$\underline{H^3 = 8,\ H^2 L = 4,\ H L^2 = L^3 = 0}$$

$$D_0 = -(D_1 + \cdots + D_6)$$

$$D_0 = -4D_3 = -4D_4 = -4D_5$$

$$D_0 = -8D_1 - 4D_6$$

$$D_0 = -8D_2 - 4D_6$$

$$D_3 = D_4 = D_5 = H$$

$$D_1 = D_2 = L$$

$$D_6 = H - 2L.$$

$$H, L \qquad \underline{h'' = 2}.$$

M a CY hypersurface in $\mathbb{P}_\nabla$.

$$P(x) = \sum_m{}' \, c_m x^m - c_0 Q \qquad Q = x_1 x_2 x_3 x_4 x_5.$$

$m \in \Delta$.
$m \neq (11111)$

$$\varpi(c,D) = \frac{\prod_m{}' \, \Gamma(D_m+1)}{\Gamma(-D_0+1)} \sum_{\gamma \in V_\nabla} \frac{\Gamma(-\gamma\cdot D_0 - D_0 + 1)}{\prod_m{}' \, \Gamma(\gamma\cdot D_m + D_m + 1)} \, c^{\gamma + D}$$

$$c^{\gamma+D} = \prod_{m\in\Delta} c_m^{\gamma_m + D_m}$$

D_m are the toric divisors of $\mathbb{P}_\nabla$

$D_0 = -\sum_m{}' \, D_m$, sum is over γ in the Mori cone of $\mathbb{P}_\nabla$.

generators of Mori cone.

$h = (-4, 0, 0, 1, 1, 1, 1)$ $\qquad$ $\ell = (0, 1, 1, 0, 0, 0, -2)$

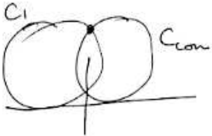
$h \cdot H = 1 \quad h \cdot L = 0$

$\ell \cdot H = 0 \quad \ell \cdot L = 1$

$$\lambda = c^h = \frac{c_3 c_4 c_5 c_6}{c_0^4} = -\frac{2\phi}{(8\psi)^4}$$

$$\mu = c^\ell = \frac{c_1 c_2}{c_6^2} = \frac{1}{(2\phi)^2}$$

$\phi^2=1$



$\phi^2=1 \quad \mathbb{P}^5[4,2] \simeq \mathbb{P}^4_{(11222)}[8]$

$\mathbb{P}^5[4,2]_{\text{conifold}}$.

R_1 degenerates $\quad (1-a_pT+p^3T^2)$

$\wr$

$\Gamma_0[16]$.

$\mathbb{P}^5_{\mathbb{F}}[2,4]$

$$y_0^2+y_1^2+y_2^2+y_3^2 = 7y_4y_5$$

$$y_4^4+y_5^4 = y_0y_1y_2y_3$$

$$y_0 = x_1^4 - x_2^4$$

$$(x_1^4-x_2^4)^2 + x_3^4 + x_4^4 + x_5^4 - 8\ell\, x_1x_2x_3x_4x_5 = 0$$

$$y_0 = x_1^4 - x_2^4$$

$$y_1 = x_3^2 \qquad y_4 = x_1\sqrt{x_3x_4x_5}$$

$$y_2 = x_4^2 \qquad y_5 = 2x_2\sqrt{x_3x_4x_5}$$

$$y_3 = x_5^2$$

$$y_0^2+y_1^2+y_2^2+y_3^2 = 8\ell\, y_4y_5$$

$$y_4^4+y_5^4 = (x_1^4-x_2^4)\,x_3^2x_4^2x_5^2$$

$$= y_0y_1y_2y_3$$