The numbers of points are determined by the periods

$$t = \frac{\varpi_1}{\varpi_0}; \quad q = e^{2\pi i t}$$

$$y_{ttt} = \frac{1}{\varpi_0^2} \frac{1}{(1-\psi^5)} (\frac{2\pi i}{5})^3 (\frac{d\psi}{dt})^3 = 5 + \sum_{k=1}^{\infty} \frac{n_k k^3 q^k}{1-q^k}$$

$$\zeta(T,\psi) = \exp\left(\sum_{k=1}^{\infty} \frac{N_k(\psi)T^k}{k}\right)$$

$$\zeta = \frac{\text{Numerator of degree } b^3 = 2h^{2,1}+2}{\text{Denominator of degree } 2h^{1,1}+2}$$

$$h^{p,q} = \begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{1,1} & & 0 & \\ 1 & & h^{2,1} & & h^{2,1} & & 1 \\ & 0 & & h^{1,1} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array}$$

Is $\zeta(\mathcal{M}) = 1/\zeta(\mathcal{W})$?

For the quintic, the numerator depends on ψ, but the denominator is always

$$(1-T)(1-pT)(1-p^2T)(1-p^3T)$$

and does not depend on the Kähler class parameters.

$$\zeta_{\mathcal{M}}(T,\psi) = \frac{R_1(T,\psi)R_A(pT,\psi)^{20}R_B(pT,\psi)^{30}}{(1-T)(1-pT)(1-p^2T)(1-p^3T)}$$

$$R_1 = 1 + aT + bpT^2 + ap^3T^3 + p^6T^4$$

$$\zeta_{\mathcal{W}}(T,\psi) = \frac{R_1(T,\psi)}{(1-T)(1-pT)^{101}(1-p^2T)^{101}(1-p^3T)}$$

$$N = N_0 + \sum_{\mathbf{v}} N_{\mathbf{v}} \qquad \Rightarrow \qquad \zeta = \zeta_0 \prod_{\mathbf{v}} \zeta_{\mathbf{v}}$$

$$\zeta_0 = \frac{R_0}{(1-T)(1-pT)(1-p^2T)(1-p^3T)}$$

ρ is the smallest $1, 2$ or 4 such that $5 | p^\rho - 1$

$$\begin{array}{c} \mathbf{v} \\ (4,1,0,0,0) \\ (3,2,0,0,0) \\ (3,1,1,0,0) \\ (2,2,1,0,0) \end{array}$$

$$\zeta_{(4,1,0,0,0)}\zeta_{(3,2,0,0,0)} = R_A(p^\rho T^\rho, \psi)^{1/\rho}$$

$$\zeta_{(3,1,1,0,0)}\zeta_{(2,2,1,0,0)} = R_B(p^\rho T^\rho, \psi)^{1/\rho}$$

$$F(a_{\mathbf{v}}, b_{\mathbf{v}}, c_{\mathbf{v}}; \psi^{-5}) \qquad a + b = c$$

$$\int dx\, x^{-\alpha/5}(1-x)^{-\beta/5}(1-\frac{x}{\psi^5})^{-(1-\beta/5)} = \int \frac{dx}{y}$$

$$\mathcal{E}_{\alpha\beta}:\ y^5 = x^\alpha(1-x)^\beta(1-\frac{x}{\psi^5})^{5-\beta}$$

	$\mathbf{v}$	(a,b,c)	(α,β)
A	$(4,1,0,0,0)$	$(2/5,3/5,1)$	$(2,3)$
	$(3,2,0,0,0)$	$(1/5,4/5,1)$	$(1,4)$
B	$(3,1,1,0,0)$	$\vdots$	$\vdots$
	$(2,2,1,0,0)$		

$$Z_A(u) = \frac{R_A(u)^2}{(1-u)(1-pu)}$$

same with B

$$\zeta = \zeta_0 \prod_{\mathbf{v}} \zeta_{\mathbf{v}} = \frac{R_0}{(1-T)(1-pT)(1-p^2T)(1-p^3T)} R_A(p^\rho T^\rho, \psi)^{20/\rho} R_B(p^\rho T^\rho, \psi)^{30/\rho}$$

When $\psi^5 = 1$, there are 125 nodes

$$\zeta(T, \psi^5 = 1) = \frac{(1 - \epsilon pT)(1 - a_p T + p^3 T^2)(1 - (pT)^\rho)^{100/\rho}}{(1 - T)(1 - pT)(1 - p^2 T)(1 - p^3 T)[1 - (p^2 T)^\rho]^{24/\rho}}$$

where $\epsilon = \left(\frac{p}{5}\right)$ and a_p is the p-th coefficient in the q-expansion of the wight 4 modular form for $\Gamma_0(25)$.

Periods are associated to the monomials Q and 100 others

$$\mathcal{I} = \left(\frac{\partial P}{\partial x_i}\right); \qquad \frac{\partial P}{\partial x_4} = 5x_4^4 - 5\psi x_1 x_2 x_3 x_5$$

$$x_1 x_2^2 x_3^3 x_4^4 \simeq \psi x_1^2 x_2^3 x_3^4 x_5 \simeq \ldots \simeq \psi^5 x_1 x_2^2 x_3^3 x_4^4$$

There are

$$\left.\begin{array}{l}125 \text{ nodes} \\ 101 \text{ parameters}\end{array}\right\} \Rightarrow \; 24 \text{ relations}$$

Each relation gives rise to a cycle

If we expand 5-adically,

$$R_1(T,\psi) = (1-T)(1-pT)(1-p^2T)(1-p^3T) + O(5^2)$$

$$R_A^{20} R_B^{30} = (1-pT)^{100}(1-p^2T)^{100} + O(5^2)$$

$$\text{so } \zeta_{\mathcal{M}} = \frac{1}{\zeta_{\mathcal{W}}} + O(5^2)$$

$$\frac{y_{ttt}}{5} = 1 + \frac{1}{5}\sum_{k=1}^{\infty} \frac{k^3 n_k q^k}{1-q^k} = 1 + O(5^2)$$

$$\text{since } 5^3 | k^3 n_k$$