$$\mathcal{L}\varpi(\lambda,\epsilon) = \epsilon^4\lambda^\epsilon$$

$$\varpi_0 = f_0$$

$$\varpi_1 = f_0 \log\lambda + f_1$$

$$\varpi_2 = f_0 \log^2\lambda + 2f_1\log\lambda + f_2$$

$$\varpi_3 = f_0 \log^3\lambda + 3f_1\log^2\lambda + 3f_2\log\lambda + f_3$$

$$\nu_\lambda = \{x \in \mathbb{F}_p^5 | P(x) = 0\};\ \ N_\lambda = \frac{\nu_\lambda - 1}{p-1}$$

$$\nu = \nu_0 + \nu_1 p + \nu_2 p^2 + \nu_3 p^3 + \nu_4 p^4$$

$$\nu_0 =^{[p/5]} f_0(\lambda)$$

We chose

$$A_n(\epsilon) \text{ with } A_n(0) = a_n = \frac{(5n)!}{n!^5}$$

But we also could have chosen

$$\tilde{A}_n(\epsilon) = h(\epsilon)A_n(\epsilon) \text{ with } h(0) = 1$$

$$h(\epsilon) = 1 + h_1\epsilon + \frac{1}{2!}h_2\epsilon^2 + \ldots$$

$$\begin{pmatrix} \varpi_0 \\ \varpi_1 \\ \varpi_2 \\ \varpi_3 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 0 \\ h_1 & 1 & 0 & 0 \\ h_2 & 2h_1 & 1 & 0 \\ h_3 & 3h_2 & 3h_1 & 1 \end{pmatrix} \begin{pmatrix} \varpi_0 \\ \varpi_1 \\ \varpi_2 \\ \varpi_3 \end{pmatrix}$$

$$\nu(\lambda) = \sum_{x \in \mathbb{F}_p^5} (1 - P(x)^{p-1}) + O(p)$$

$$\nu(\lambda) = \sum_{x \in \mathbb{F}_p^5} (1 - P(x)^{p(p-1)}) + O(p^2)$$

$$p^{p-1} = 1 + O(p) \qquad p^{p(p-1)} = 1 + O(p^2)$$

$$\nu(\lambda) =^{[2p/5]} f_0(\lambda^p) + p^{[2p/5]} f_1'(\lambda^p) + O(p^2)$$

$$\text{where } f_1'(\lambda^p) = (\theta f)(\lambda^p); \qquad \theta = \lambda \frac{d}{d\lambda}$$

$$\text{mod } p: \; \lambda = \lambda + p \quad \text{so} \quad f(\lambda + p) = f(\lambda) + O(p)$$

$$(\lambda + p)^p = \lambda^p + O(p^2)$$

$$\text{Teich}(\lambda) = \lim_{n\to\infty} \lambda^{p^n}$$

For higher orders, we try to guess the result:

$$\nu(\lambda) = (1 + Ap^2)^{[3p/5]} f_0(\lambda^{p^2}) + p(1 + Bp)^{[3p/5]} f_1'(\lambda^{p^2}) + Cp^{2^{[3p/5]}} f_2''(\lambda^{p^2}) + O(p^3)$$

$$\nu = {}^{[p]}f_0(\lambda^{p^4}) + \tfrac{p}{1-p}{}^{[p]}f_1'(\lambda^{p^4}) + \tfrac{1}{2!}(\tfrac{p}{1-p})^{2\,[p]}f_2''(\lambda^{p^4}) + \ldots +$$

$$+\tfrac{1}{4!}(\tfrac{p}{1-p})^{4\,[p]}f_4''''(\lambda^{p^4}) + h_3(\tfrac{p}{1-p})^3\{{}^{[p]}f_0''' + (\tfrac{p}{1-p})^{[p]}f_1'''\} + O(p^5)$$

We can get rid of the last term with a change of basis

$$h(\epsilon) = 1 + \frac{1}{3!}h_3\epsilon^3 + O(\epsilon^5)$$

$$\mathcal{L}\varpi(\lambda,\epsilon) = \epsilon^4\lambda^\epsilon \qquad \varpi = \sum \frac{1}{n!}\epsilon^n\varpi_n$$

$$\frac{a_{rp}}{a_r} = \frac{\Gamma_p(5rp+1)}{\Gamma_p^5(rp+1)} = h(rp)$$

$$\nu_\lambda = \sum_{m=0}^{p-1} \frac{a_{m(1+p+p^2+p^3+p^4)}}{a_{m(1+p+p^2+p^3)}} \lambda^{mp^4} + O(p^5) = \sum_{m=0}^{p-1} \beta_m \mathrm{Teich}(\lambda)^m$$

$$\text{where } \beta_m = \lim_{n\to\infty} \frac{a_{m(1+p+\ldots+p^{n+1})}}{a_{m(1+p+\ldots+p^n)}}$$

Lower dimensional examples:

$$\mathbb{P}_1[2]: \quad x_1^2 + x_2^2 - 2\psi x_1 x_2 = 0; \text{ is singular for } \psi^2 = 1$$

$$\lambda = \frac{1}{(2\psi)^2} \qquad \mathcal{L} = \theta - 2\lambda(2\theta + 1)$$

$$\varpi_0 = f_0 = \frac{1}{\sqrt{1-4\lambda}} = \sum_{n=0}^{\infty} \frac{(2n)!}{n!^2} \lambda^n$$

$$\varpi_1 = f_0 \log \lambda + f_1; \quad f_1 = 2 \sum_{n=1}^{\infty} \frac{(2n)!}{n!^2} (\sigma_{2n} - \sigma_n) \lambda^n; \quad \sigma_n = \sum_{k=1}^{n} \frac{1}{k}$$

We can calculate N_λ from f_0 and f_1.

$$\mathbb{P}_0[1]: \qquad P(x) = x - \psi x = (1-\psi)x$$

$$\nu = \begin{cases} 1 & \text{if } \psi \neq 1 \\ p & \text{if } \psi = 1 \end{cases}$$

$$\varpi_0 = \sum \frac{n!}{n!} \lambda^n = \sum \lambda^n = \frac{1}{1-\lambda} = f_0$$

$${}^{[p]}f_0 = \sum_1^{p-1} \lambda^n = \begin{cases} \frac{\lambda^p - 1}{\lambda - 1} = 1 & \text{if } \lambda \neq 1 \\ p & \text{if } \lambda = 1 \end{cases}$$

Dwork's character

$$\Theta(x) = \begin{cases} \exp[\pi(x - x^p)] \\ E(\pi x) \end{cases}$$

$$\Theta(x + y) = \Theta(x)\Theta(y)$$

$$\sum_{y \in \mathbb{F}_p} \Theta(yP(x)) = \begin{cases} p & \text{if } P(x) = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\Theta(yP) = \Theta(y(\sum x_i^5 - 5\psi \prod x_i)) = \Theta(-5\psi y \prod x_i) \prod_i \Theta(y x_i^5)$$

$$G_n = \sum_{x \in \mathbb{F}_p^\star} \Theta(x) (\text{Teich } x)^n$$

$$g_n(y) = \sum_{x \in \mathbb{F}_p^\star} \Theta(y x^5) (\text{Teich } x)^n$$

$$\Theta(x) = \frac{1}{p-1} \sum_{m=0}^{p-2} G_{-m} (\text{Teich } x)^m$$

$$\sum_{x \in \mathbb{F}_p^\star} (\text{Teich } x)^n = \begin{cases} 0 & \text{if } p-1 \not| \, n \\ p-1 & \text{otherwise} \end{cases}$$

If $5 \not| \, p-1$, then

$$\nu = 1 + p^4 + \sum_{m=1}^{p-2} \frac{G_m^5}{G_{5m}} (\text{Teich } \lambda)^{-m}$$

On $\mathcal{M}$ we have an action of G

$$x_i \to \alpha^{n_i} x_i; \qquad \alpha^5 = 1, \quad \sum n_i = 0 \pmod 5$$

The mirror is $\mathcal{W} = \widehat{\mathcal{M}/G}$ (Resolving the singularities of $\mathcal{M}/G$).

$$h^{1,1}(\mathcal{W}) = h^{2,1}(\mathcal{M}) = 101$$

$$h^{2,1}(\mathcal{W}) = h^{1,1}(\mathcal{M}) = 1$$

If $5|p-1$, then

$$\nu = {}^{[p]}f_0 + \frac{p}{1-p}{}^{[p]}f_1' + \ldots + p\sum_{\mathbf{v}} \frac{\gamma_{\mathbf{v}}}{\prod_i (v_i k)!}{}^{[p]}{}_2F_1(a_{\mathbf{v}}, b_{\mathbf{v}}, c_{\mathbf{v}}; \psi^{-5}) + O(p^2)$$

where $k = (p-1)/5$