

$$\underline{\text{Def}} \quad \mathbb{C}_{\text{exp}} = (\mathbb{C}, +, \cdot, \exp, 0, 1)$$

$$\mathcal{L} (+, \cdot, E, 0, 1)$$

$$E^{\text{exp}} = \exp$$

$$\exp(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!}$$

$$\underline{\text{Def}} \quad \mathbb{R}_{\text{exp}} = (\mathbb{R}, +, \cdot, \exp, 0, 1)$$

Thm (Wilkie)

$\text{Th}(\mathbb{R}_{\text{exp}})$ is
model complete

and \mathcal{O} -minimal

Prop Th(C_{exp})
is undecidable.

Pf $\varphi(x) ::=$

$$\forall z [E(z) = 1]$$

$$\rightarrow E(xz) = 1]$$

$$\varphi(C_{exp}) = \mathbb{Z}_1$$

From a decision

procedure for $Th(C_{exp})$

We obtain a
decision procedure
for $\mathcal{T}(\mathbb{Z}, +, \cdot, 0, 1)$

Impossible by

Gödel. //

We work in
 $L_{\omega_1, \omega}(Q)$

We allow countable

conjunction, countable

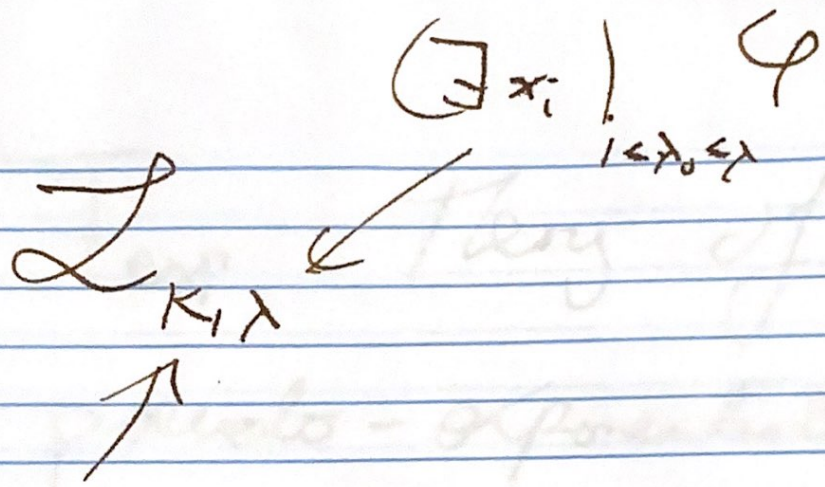
disjunction, usual

first operators,

" $\exists x$ " ~ "there are

uncountable many x

s.t. ..."



conjecture of $< K$ Fontaine

closed field

A(F) : atoms

for algebraically

closed fields of

characteristic zero

T_{exp} Theory of pseudo-exponentiation

- ELA "exponential, logarithmic, algebraically closed field"

ACF₀: axioms
for algebraically
closed fields of
characteristic zero.

eg. $\forall x \forall y \forall z$

$$x \cdot (y+z) = x \cdot y + x \cdot z$$

- $1+1 \neq 0$

$\neg (1+1=0)$

$1+1+1 \neq 0$

- $\forall a_0 \forall a_1 \dots \forall a_{n-1}$

$$\exists x \quad x^n + a_{n-1}x^{n-1} + \dots + a_0 = 0$$

$$\cdot E(0) = 1$$

$$\cdot \forall x \forall y \quad E(x+y) = E(x) \cdot E(y)$$

$$\cdot \forall y [y=0 \overset{\text{or}}{\vee} \downarrow$$

$$\exists x \quad E(x) = y]$$

SK : standard kernel

$$\text{ker } E \cong \mathbb{Z}$$

~~$$\forall x [E(x) = 1$$~~

~~$$\forall n \in \mathbb{Z}$$~~

$$\cancel{\forall x [E(x) = 1]}$$

$$\exists \omega [\forall x E(x) = 1]$$

$$\rightarrow \left. \begin{array}{l} \bigvee_{n \in \mathbb{Z}} \underbrace{\omega + \dots + \omega}_{n \text{-times}} = x \end{array} \right\}$$

SC Schanuel's
Conjecture

$$\forall \alpha_1, \dots, \forall \alpha_n$$

$$\left[\text{if } \dim_{\mathbb{Q}} (\mathbb{Q}\alpha_1 + \dots + \mathbb{Q}\alpha_n) = n, \right.$$

then $\text{tr deg}_{\mathbb{Q}} \mathbb{Q}(\vec{\alpha}, \vec{E}(\alpha_1))$

$$\geq n$$

Example if ω is

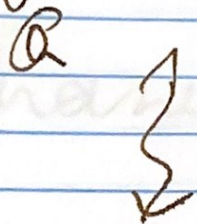
$$E(\omega) = 1, \omega \neq 0$$

\mathbb{R} ω is transcendental

$$\alpha_1 = \omega$$

$$\text{tr deg}_{\mathbb{Q}} \mathbb{Q}(\omega, E(\omega)) \geq 1$$

$$\# \text{ of } \mathbb{Q}(\alpha, E(\alpha)) \geq n$$



$$\Delta \rightarrow \left(\begin{array}{l} f_1(\alpha, E(\alpha)) = 0 \text{ \& } \\ \dots \text{ \& } f_m(\alpha, E(\alpha)) = 0 \end{array} \right)$$

$$f_1, \dots, f_m \in \mathbb{Q}[x_1, \dots, x_n, y_1, \dots, y_n]$$

$$\dim(V(\vec{f})) < n$$

518ac

Schanuel's

Reverse Conjecture:

If $K = (K, +, -, E, 0, 1)$

is a countable

field satisfying $E \perp A + S/K$

+ SC , then

\exists embedding

$K \hookrightarrow \mathbb{C}_{exp}$

$$\text{If } X \subseteq \mathbb{G}_a^g \times \mathbb{G}_m^g$$

additive group multiplicative

is an algebraic

variety (irreducible)

and X is

normal

and additively and
multiplicatively free

then $\exists a \in (a_1, \dots, a_g)$

$$\in \mathbb{G}_a^g(K)$$

$$(a, E(a)) \in X(K)$$

additively free:

$$\nexists (a_1, \dots, a_g) \in \mathbb{Z}^g \setminus \{(0, \dots, 0)\}$$

$$l x_1 + \dots + l x_g \text{ is constant on } X$$

multiplicatively free:

no $(l_1, \dots, l_g) \in \mathbb{Z}^g \setminus \{0\}$

$y_1^{l_1}, \dots, y_g^{l_g}$ is constant
on X .

IF $M = (M_{ij})$

$\in M_{g \times m}(\mathbb{Z})$

$$\overline{\Phi}_M: \mathbb{G}_a^g + \mathbb{G}_a^n \rightarrow \mathbb{G}_a^m + \mathbb{G}_a^n$$

$$(x_1, \dots, x_g, y_1, \dots, y_g)$$

$$\mapsto \left(\sum M_{1j} x_j, \dots, \sum M_{gj} x_j \right)$$

$$\left(\prod_{j=1}^{M_1} y_j, \dots, \prod_{j=1}^{M_n} y_j \right)$$

Notend:

$$\dim \mathbb{F}_M^X \geq \text{rk } M.$$

CCP: countable
dense.