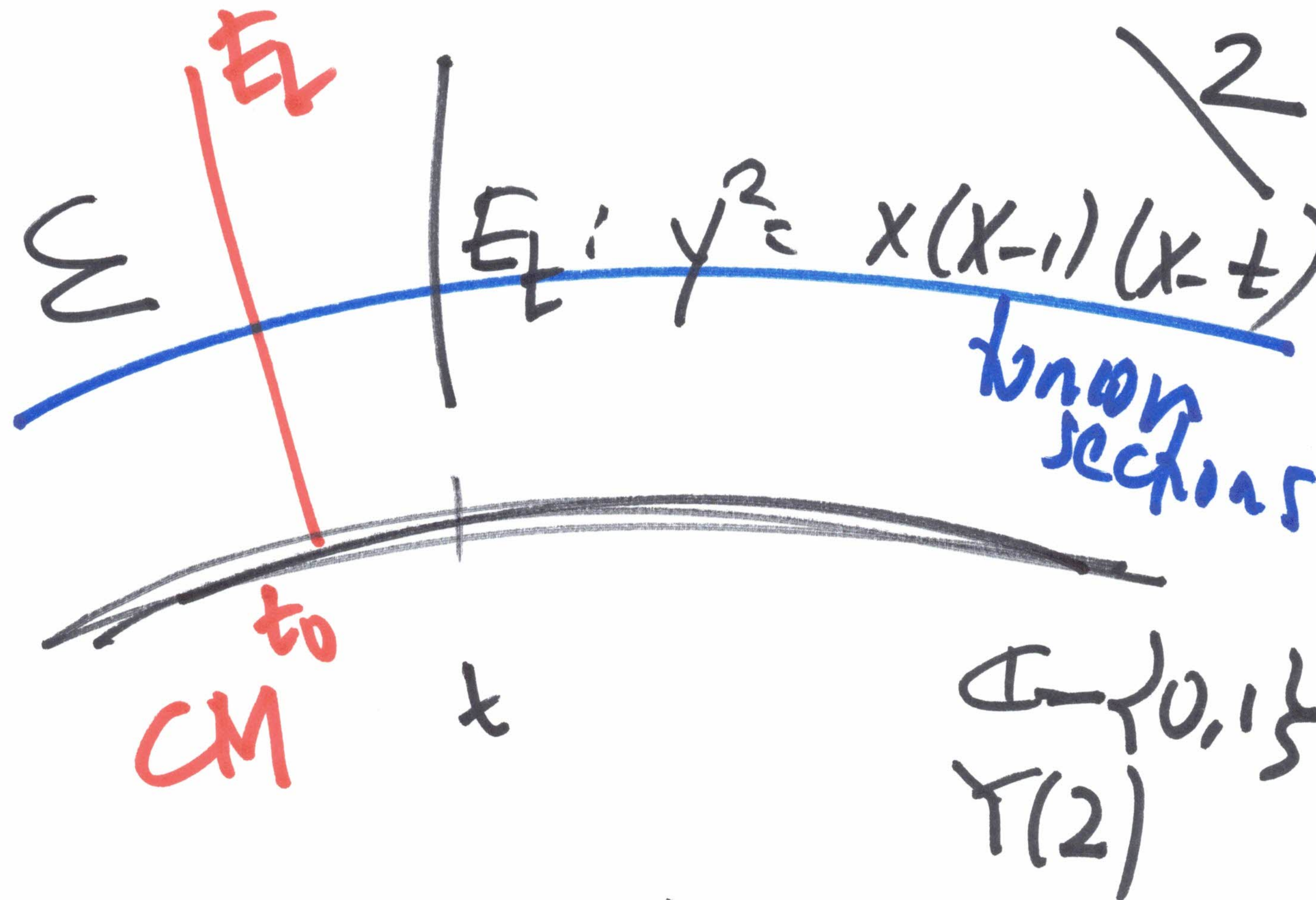
ZP

CIT : $(\mathbb{G}_m^n = X$, torsion cosets$)$.

$(\mathrm{Ab\,Var}$, torsion cosets$)$

$(Y(1)^n$, special subvar$)$

$(\mathcal{A}_g, \mathrm{Sh}$, special subvars$)$.



Simplest MSV.

$(\Sigma,$ special subvs$)$

$=$

$\{(t,x,y):$

General Picture for ZP

$$(X, \mathcal{S})$$

MSV — collection of sp subv.

Let $V \subset X$.

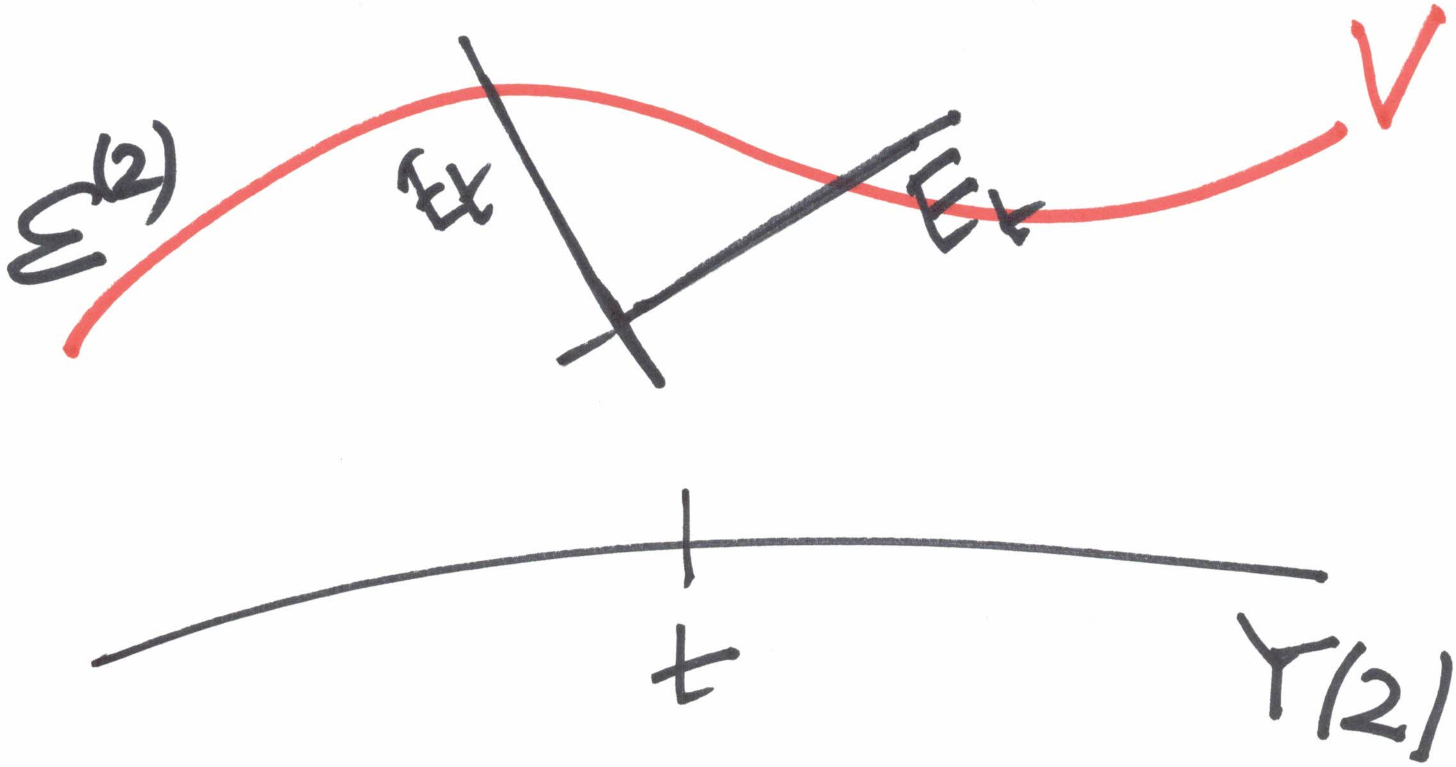
<u>Definition</u>: A subvariety $A \subset V$ is <u>atypical</u> (of V in X) if $A \subset_{cpt} V \cap S$ some $S \in \mathcal{S}$ and

$$\dim A > \dim V + \dim S - \dim X$$

ZP Conjecture:

With X MSV, $V \subset X$ then the UA of all atypical components is a finite union.

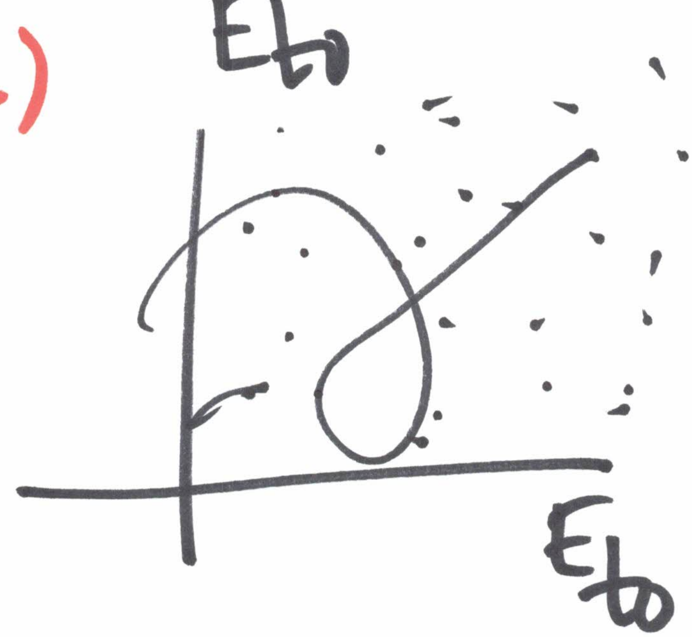
RMM



$\dim \mathcal{E}^{(2)} = 3.$

$V \subset \mathcal{E}^{(2)}$

$V \subset E_{t_0}^2$



Maner Zammer:

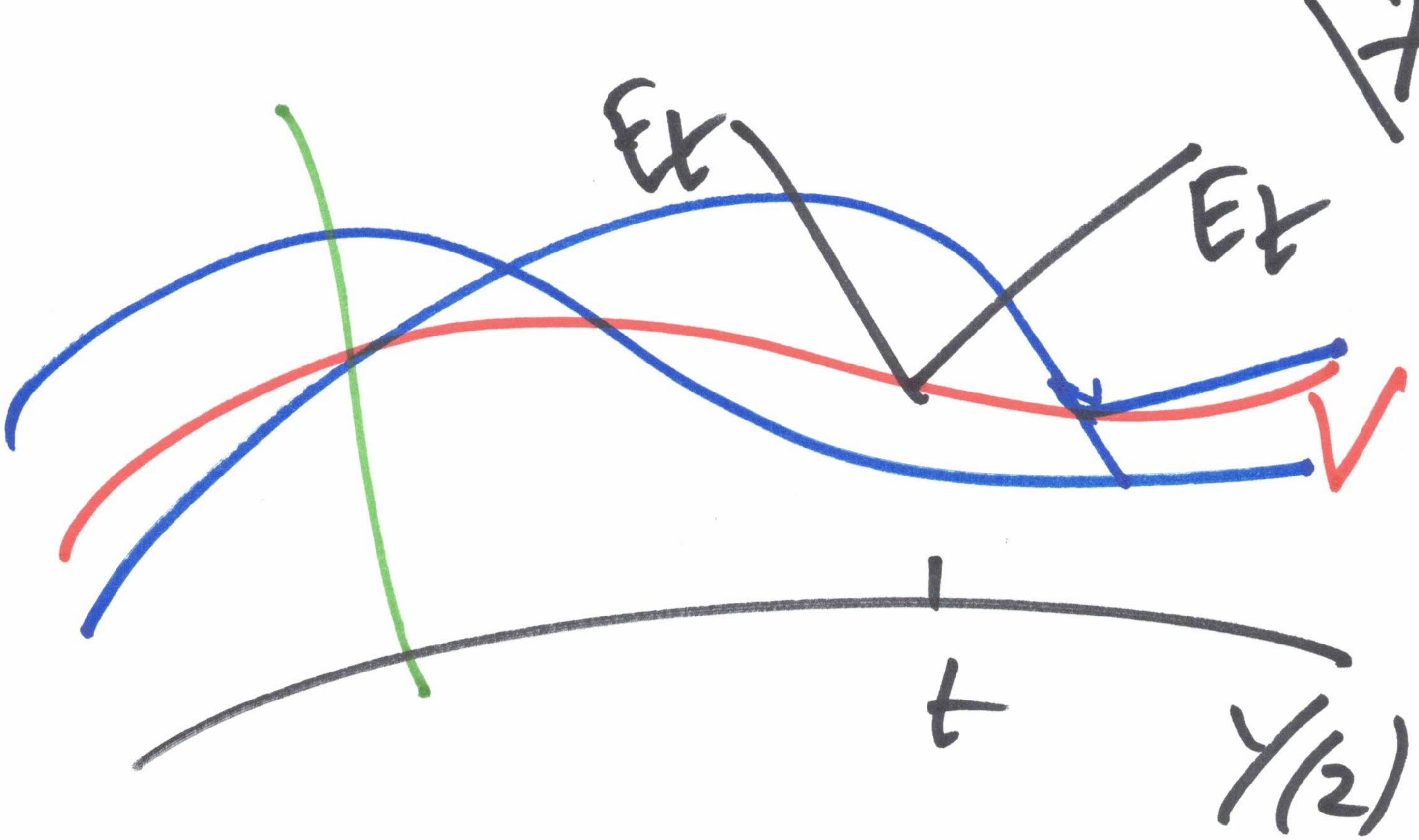
$$V = \{(t, \sqrt{2(2-t)}, \sqrt{6(3-t)})\}$$

pt on E_t with x coord $= 2$

x coord $= 3$.

⋮

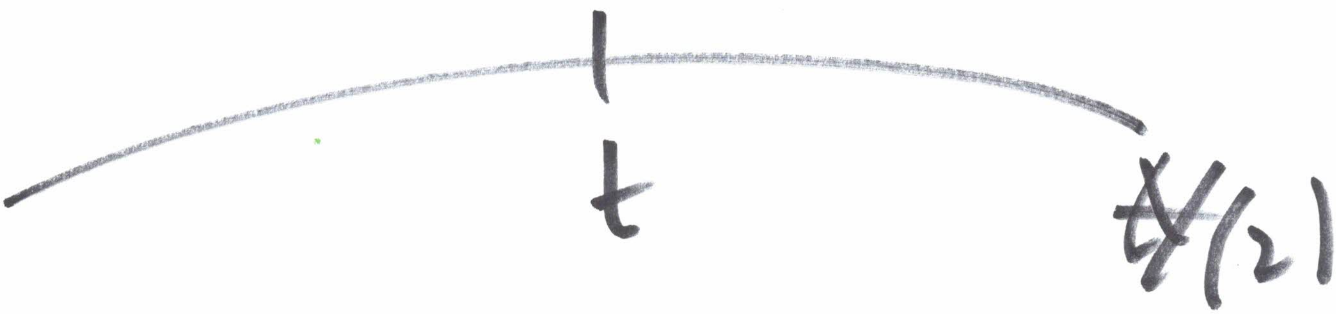
Gao-Rabegger



<u>Theorem</u>: If V is not identically torsion, then $\{t\} \in V$ for which point torsion is <u>finite</u>.

Points on V where t parameter is special and $(x, y) \in V$ are l. dep.
<u>Bombieri</u>.

$\mathcal{E}^{(3)}$



<u>Theorem</u> (Barroero-Capuano)

$V \subset \mathcal{E}^{(n)}$, and $P_1(t), \ldots, P_n(t)$ are generically lin. indpt $/\mathbb{Z}$. Then $\exists$ only finitely many t: $P_1(t), \ldots P_n(t)$ satisfy <u>two</u> indept linear relation over $\mathbb{Z}$.

$V \subset E_{t_0}^{3}$

Analogue of ZP

$X = \mathbb{A}^n/\mathbb{C}$

Special subvarieties:

all $T/\overline{\mathbb{Q}}$.

$V/\mathbb{C} \subset X \ldots$

Theorem: (Chatzidakis, Ghioca, Masser, Maurin)

"ZP" holds here.

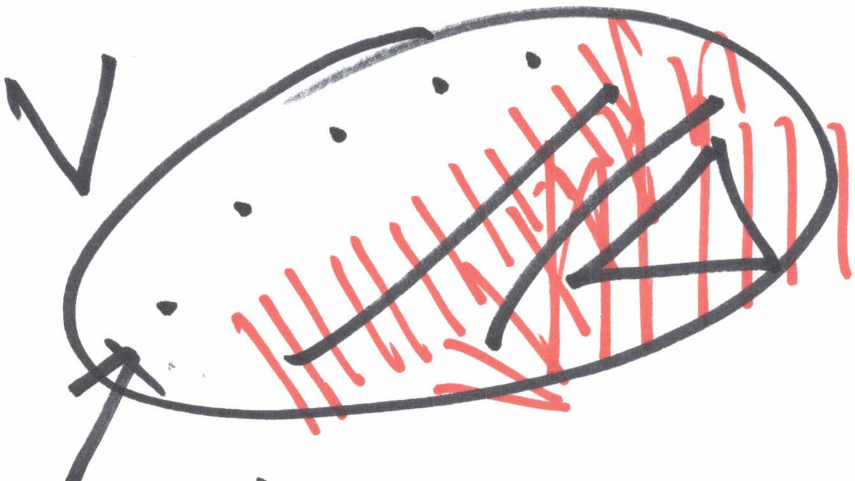
$V \subset Y(1)^3$.
curve

Previous theorem implies: if $V/\mathbb{C}$ is not contained in any hypersurface $/\overline{\mathbb{Q}}$ then V satisfies ZP.

<u>Theorem</u>: ZP holds also for any curve $V/\mathbb{C}$ not defined over $\overline{\mathbb{Q}}$.

$\mathbb{C}^n_{\mathrm{rm}}$ case.

$$V \subset \mathbb{C}^n_{\mathrm{rm}}$$



<u>Theorem</u>:
(Habegger)

Multiplicative dependence & singular moduli

$\sigma_1, \ldots, \sigma_n$ are

$V \subset Y(1)^n \times \mathbb{G}_m^n$

$= \{(x_1, \ldots x_n, x_1, \ldots, x_n)\}.$

ZP $\Rightarrow$

ZP ⇒ UZP \13

Curve $V \subset \mathbb{T}(1)^3$

Thm: ~~UZP:~~ Assume ZP ~~for~~ for all power of $\mathbb{T}(1)^n$

Given $d \geq 1$ then there is a number N_d such that if $V \subset \mathbb{T}(1)^3$ is a curve of degree d then # atypical pts is at most $\underline{N_d}$.

$E_t : y^2 = x(x-1)(x-t).$