LUBIN-TATE SPACE: OLD AND NEW QUESTIONS
ARIZONA WINTER SCHOOL 2019

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Course outline

The Lubin-Tate formal groups and their corresponding deformation spaces have played many roles in mathematics, from their initial conception as an explicit means of realizing local class field theory, through their role in the local Langlands correspondence, and the many appearances they make in homotopy theory. For the most part the questions of importance in algebraic topology are very different from those of interest in number theory. One goal of this course is to lay out these diverse questions in the hopes of bridging these two audiences. There are also strong theoretical demands from both the algebraic topology side and number theoretic sides and wherever possible, the theory will be supplemented with explicit formulas.

Here is a tentative outline of the lecture series

(1) Origin story
  (a) Local class field theory
  (b) Deformations
  (c) The moduli stack of commutative 1-dimensional formal groups
(2) Period maps
  (a) Periods according to Abel and Jacobi
  (b) Periods and uniformization
  (c) Cartier's magic carpet
  (d) Crystalline Dieudonne theory
  (e) Finite orbifold quotients.
(3) Drinfeld covers
  (a) Level structures according to Drinfeld
  (b) Level structures in algebraic topology
  (c) Fricke involutions and the crystalline period map
(4) Picard groups
  (a) Picard groups and Iwasawa theory
  (b) Picard groups of Lubin-Tate spaces

Projects

Here are some possible projects the participants might work on in order to become better involved with this material. Some of these represent known results and some appear to be very difficult problems.

Tangent spaces to moduli stacks. Deformation theory is often controlled by the cotangent complex of a stack. The Lubin-Tate deformation theory can be interpreted in this language. It is illuminating to work out explicitly how this works.

Algebraic approximations to period maps. The period maps are p-adic analytic maps, given by solutions to p-adic Picard-Fuchs equations. In the case of Lubin-Tate space these solutions can be made very explicit. The crystalline period map comes from a map of formal schemes, though this map does not as yet have an interpretation in terms of moduli. Approximations to this map of formal schemes can be worked out explicitly. Doing so might give some hope of thinking of the period map in terms of formal moduli.

Uniformization of orbifold quotients. The quotients of the Lubin-Tate deformation space by finite groups of automorphisms of the Lubin-Tate group are extremely useful in algebraic topology. One good project here is to work out the classification of such finite groups. While one could find out the answer with a google search, doing so by hand gives some insights into the correspondence between Galois cohomology and division algebras. Building on this it is possible to show that the finite orbifold quotient stacks of the Lubin-Tate deformation space have particularly nice uniformizations.

Picard groups of Lubin-Tate spaces. Determining the Picard group of the Lubin-Tate stack is an important problem in algebraic topology. What is known is known by a lengthy computation. Trying to come to grips with this computation and perhaps generalize it would be an interesting project. Students could expect a good payoff in learning something, though the problem still appears to be quite challenging.

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