Arizona Winter School 2017 course outline: Adic spaces

Adic spaces are non-archimedean analytic objects which were developed by Huber in the 1990s [1]. The category of adic spaces contains both formal schemes and rigid-analytic varieties as full subcategories; perfectoid spaces [3] are further examples. The central idea is that to a certain sort of topological ring A (a Huber ring) one can associate a topological space $\operatorname{Spa} A$, its adic spectrum, whose points correspond to continuous valuations on A [2]. General adic spaces are obtained by gluing together ringed spaces of the form $\operatorname{Spa} A$.

In this series of lectures we present an introduction to the theory with an emphasis on examples. Topics may include:

- The adic spectrum of a Huber ring
- The adic closed unit disc D over $\mathbf{Q}_p$, and its 5 classes of points; the closure of D in $\mathbf{A}^1$
- The adic generic fiber of a formal scheme
- The product $\operatorname{Spa} K \times \operatorname{Spa} K$, where $K = \mathbf{F}_p((t))$
- The adic space $\operatorname{Spa} W(\mathbf{O}_K)$ for a perfectoid field K, and untilts
- The perfectoid disc; universal covers of p-divisible groups and abelian varieties
- The pro-étale topology, and the locally perfectoid nature of rigid spaces
- Comparison theorems for rigid spaces

References

[1] Huber, R. *A generalization of formal schemes and rigid analytic varieties*. Math. Z. 217 (1994), no. 4, 513-551.

[2] Huber, R. *Continuous valuations*. Math. Z. 212 (1993), no. 3, 455-477.

[3] Scholze, P. *Perfectoid Spaces*. Publ. Math. Inst. Hautes tudes Sci. 116 (2012), 245-313.

[4] Scholze, P. *p-adic Hodge theory for rigid-analytic varieties*. Forum Math. Pi 1 (2013), e1, 77 pp.

[5] *Lectures on p-adic geometry*. Notes from P. Scholze's course at Berkeley in 2014. Available at `http://math.bu.edu/people/jsweinst/Math274/ScholzeLectures.pdf`.

[6] Wedhorn, T. *Adic Spaces*. Notes available at `http://math.stanford.edu/~conrad/Perfseminar/refs/wedhornadic.pdf`.

[7] Weinstein, J. Gal($\overline{\mathbf{Q}}_p/\mathbf{Q}_p$)) *as a geometric fundamental group*. Int. Math. Res. Not. (2016).